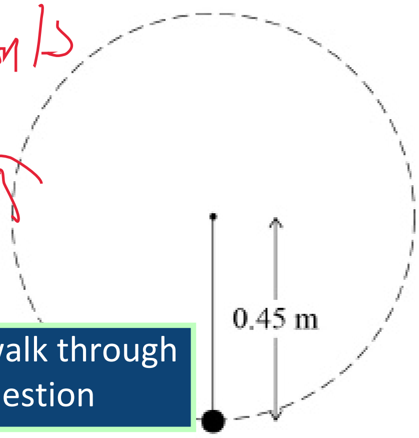
1 A bob of mass 0.50 kg is suspended from the end of a piece of string 0.45 m long.
The bob is rotated in a vertical circle at a constant rate of 120 revolutions per minute.

$v = \frac{2\pi r}{T} = 5.65$ m/s

$F = \frac{mv^2}{r} = 35.5$ N



click to see a walk through of this question

weight $= 0.5 \times 9.8$
$= 4.9$ N
which produces an upward tension of 4.9

total = 40.4

What is the tension in the string when the bob is at the bottom of the circle?

A 5.8 N ⬭

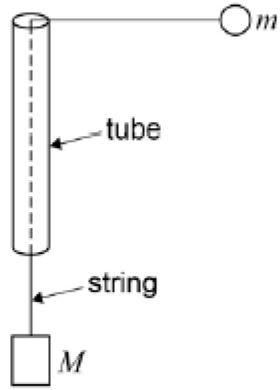
B 31 N ⬭

C 36 N ⬭

D 40 N ⬭

(Total 1 mark)

2 A string passes through a smooth thin tube. Masses m and M are attached to the ends of the string. The tube is moved so that the mass m travels in a horizontal circle of constant radius r and at constant speed v.



$\frac{mv^2}{r} = Mg$

so $M = \frac{mv^2}{rg}$

Which of the following expressions is equal to M ?

× **A** $\frac{mv^2}{2r}$

B mv^2rg

C $\frac{mv^2}{rg}$

D $\frac{mv^2g}{r}$

(Total 1 mark)

3 **Figure 1** shows a side view of an act performed by two acrobats. **Figure 2** shows the view from above.

Figure 1

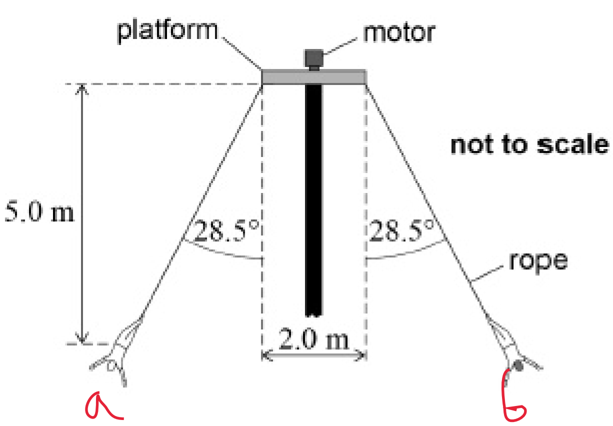
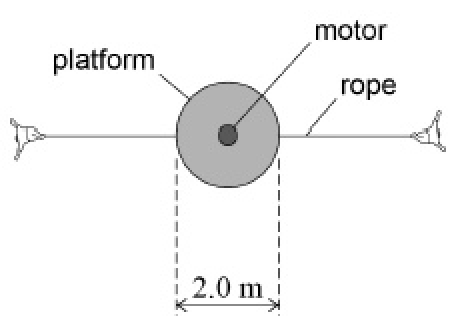


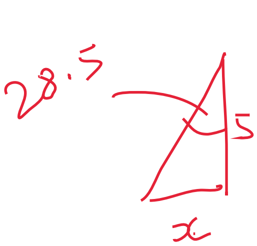
Figure 2



The acrobats, each of mass 85 kg, are suspended from ropes attached to opposite edges of a circular platform that is at the top of a vertical pole. The platform has a diameter of 2.0 m
A motor rotates the platform so that the acrobats move at a constant speed in a horizontal circle, on opposite sides of the pole.

When the period of rotation of the platform is 5.2 s, the centre of mass of each acrobat is 5.0 m below the platform and the ropes are at an angle of 28.5° to the vertical as shown in **Figure 1**.

(a) Show that the linear speed of the acrobats is about 4.5 m s^{-1}



$\tan 28.5 = \frac{x}{5} \Rightarrow x = 2.71$

$\therefore$ radius $= 2.71 + 1$

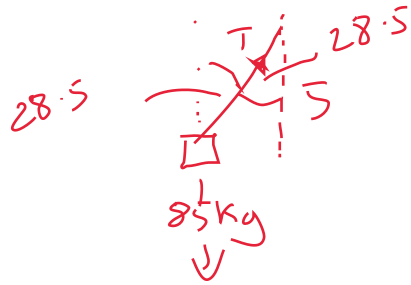
$= 3.71$

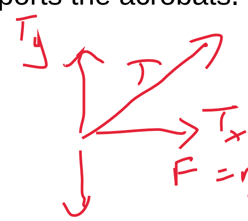
$\therefore$ circ $= 2\pi \times 3.71 = 23.34$ m in 5.2 sec

$v = 23.34 / 5.2 = 4.49 = \underline{4.5}$ m/s

(2)

(b) Determine the tension in each rope that supports the acrobats.





$F = \frac{mv^2}{r}$

$T_y = 85 \times 9.8$

$T_x = \frac{85 \times (4.5)^2}{3.71}$

$T^2 = T_y^2 + T_x^2 = 833^2 + 464^2$

$= 953$

tension = ____________________ N

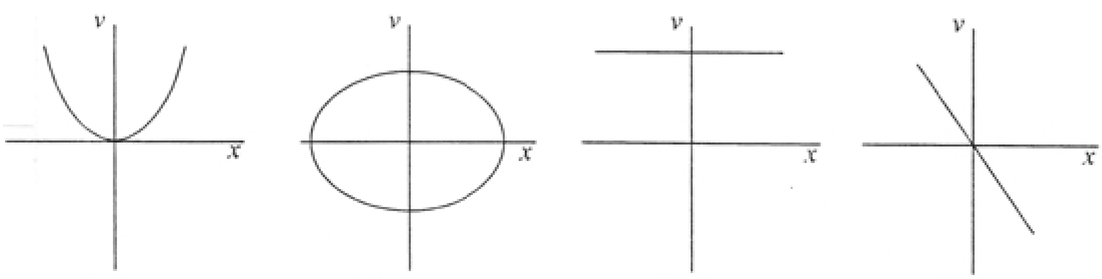
(3)

(c) Discuss the consequences for the forces acting on the pole when one acrobat has a much greater mass than the other.

- Centripetal forces on each acrobat different.
- leading to unbalanced horizontal forces on pole so perhaps it bends a little
- some more compression of the pole due to increased vertical components

(3)
(Total 8 marks)

4 Which graph shows how the velocity v of a body moving with simple harmonic motion varies with its displacement x?



A **B** **C** **D**

A ○

B ○

C ○

D ○

velocity is a vector
so not a or c
∴ should have both signs
∴ B

(Total 1 mark)

5 A body performs simple harmonic motion.

What is the phase difference between the variation of displacement with time and the variation of acceleration with time for the body?

half a cycle

A 0 ⬭

B $\frac{\pi}{4}$ rad ⬭

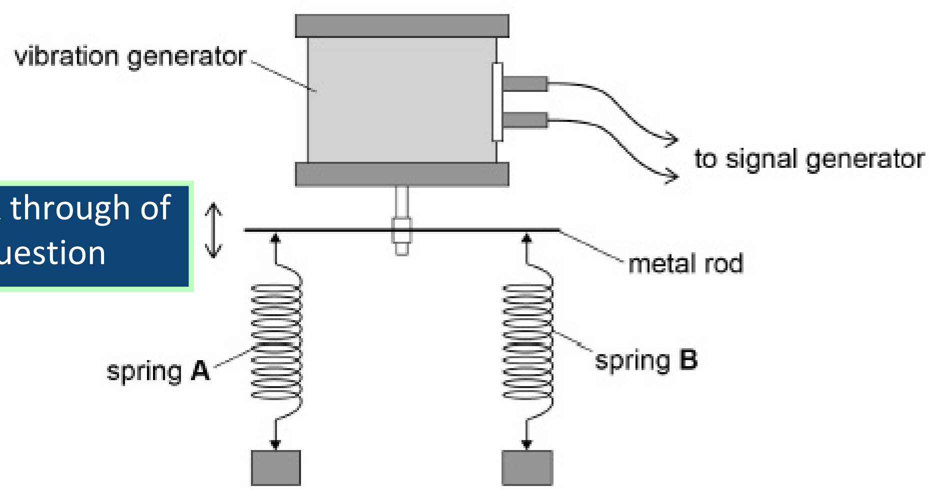
C $\frac{\pi}{2}$ rad ⬭

D π rad ⬭

(Total 1 mark)

6 A student is investigating forced vertical oscillations in springs.
Two springs, **A** and **B**, are suspended from a horizontal metal rod that is attached to a vibration generator. The stiffness of **A** is k, and the stiffness of **B** is $3k$.
Two equal masses are suspended from the springs as shown in **Figure 1**.

Figure 1



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The vibration generator is connected to a signal generator. The signal generator is used to vary the frequency of vibration of the metal rod. When the signal generator is set at 2.0 Hz, the mass attached to spring **A** oscillates with a maximum amplitude of 2.5×10^{-2} m and has a maximum kinetic energy of 54 mJ.

(a) Deduce the spring constant k for spring **A** and the mass m suspended from it.

$A = 2.5 \times 10^{-2}\,m$ $E_k = 54 \times 10^{-3}\,J$ max $f = 2\,Hz$

$v = \pm\omega\sqrt{A^2 - x^2}$ $T = 0.5\,s$

$\omega = 2\pi f$

so $v_{max} = \omega A = 2\pi f \times 2.5 \times 10^{-2} = 0.314\,m/s$

so $E_k = \frac{1}{2} m v^2$

$T = 2\pi\sqrt{\frac{m}{k}} \Rightarrow f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \Rightarrow (2\pi f)^2 \times m = k$

$\frac{2E_k}{v^2} = m \Rightarrow m = 1.1\,kg$

k = 173 N m^{-1}

m = 1.1 kg

(4)

(b) Calculate the frequency at which the mass attached to spring **B** oscillates with maximum amplitude.

$k_B = 3k_A = 519\,N\,m^{-1}$ $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$ so $f \propto \sqrt{3}$

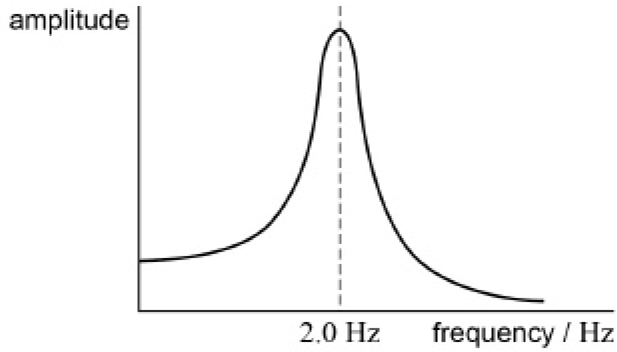
$2\sqrt{3}$

frequency = 3.5 (3.5) Hz

(2)

(c) **Figure 2** shows how the amplitude of the oscillations of the mass varies with frequency for spring **A**.

Figure 2



The investigation is repeated with the mass attached to spring **B** immersed in a beaker of oil.

A graph of the variation of the amplitude with frequency for spring **B** is different from the graph in **Figure 2**.

Explain **two** differences in the graph for spring **B**.

Difference 1 peak freq will be at 3.5 Hz since that is its natural frequency ∵ k is higher

Difference 2 max amp will be less due to damping from oil

(4)

(Total 10 marks)

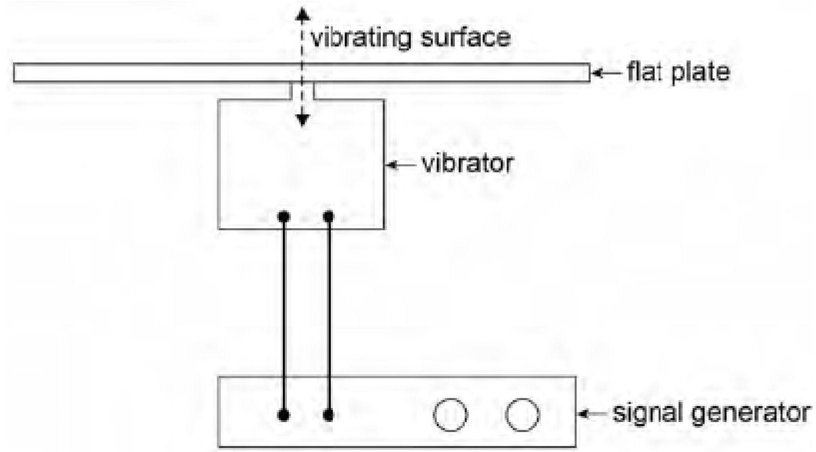
7 (a) State the conditions for simple harmonic motion.

restoring force towards center & is proportional to displacement.

(2)

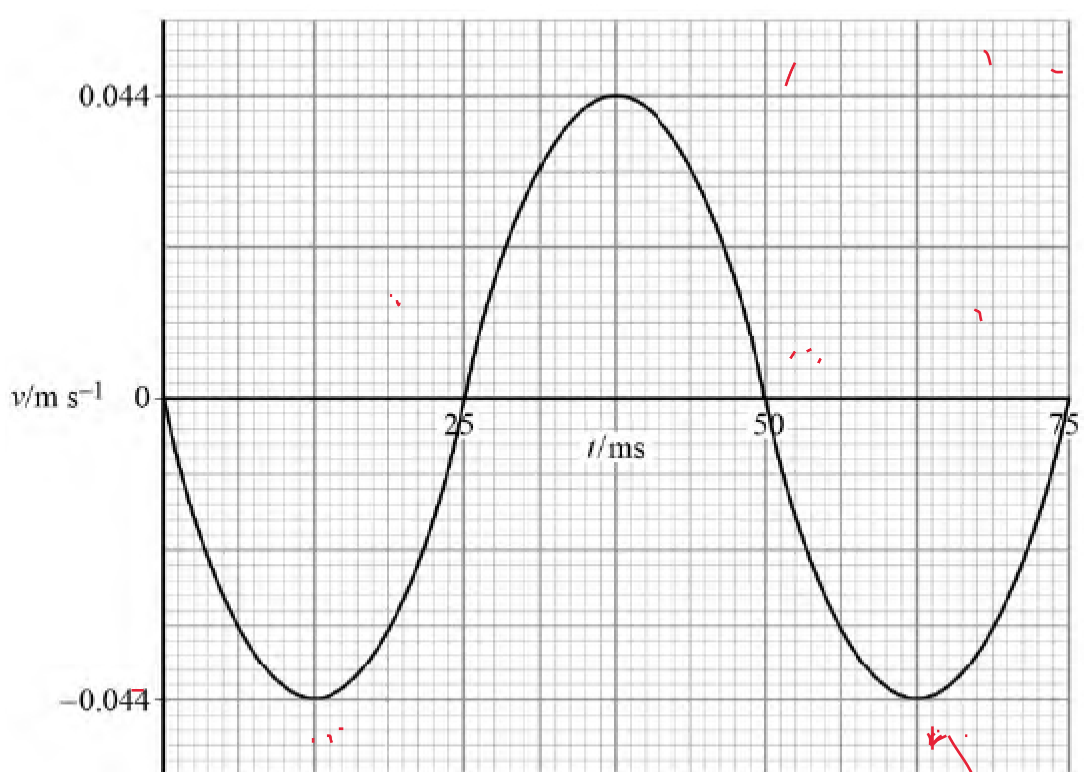
(b) A rigid flat plate is made to vibrate vertically with simple harmonic motion. The frequency of the vibration is controlled by a signal generator as shown in **Figure 1**.

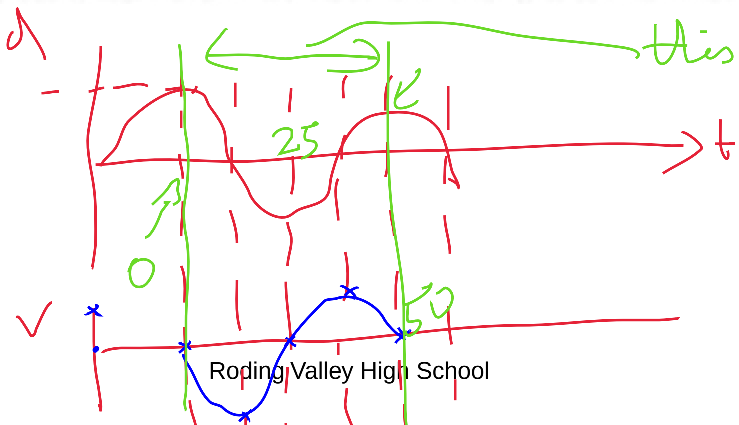
Figure 1



The velocity−time (v−t) graph for the vibrating plate at one frequency is shown in **Figure 2**.

Figure 2





this bit for part c ie cosine

Show that the maximum displacement of the plate is 3.5×10^{-4} m.

$v = \pm \omega\sqrt{A^2 - x^2}$

$0.044 = +40\pi\sqrt{A^2 - \cancel{x^2}}$

$= \omega A = 3.5 \times 10^{-4}$

$T = 50 \times 10^{-3}$ s

$\therefore f = 20$ Hz

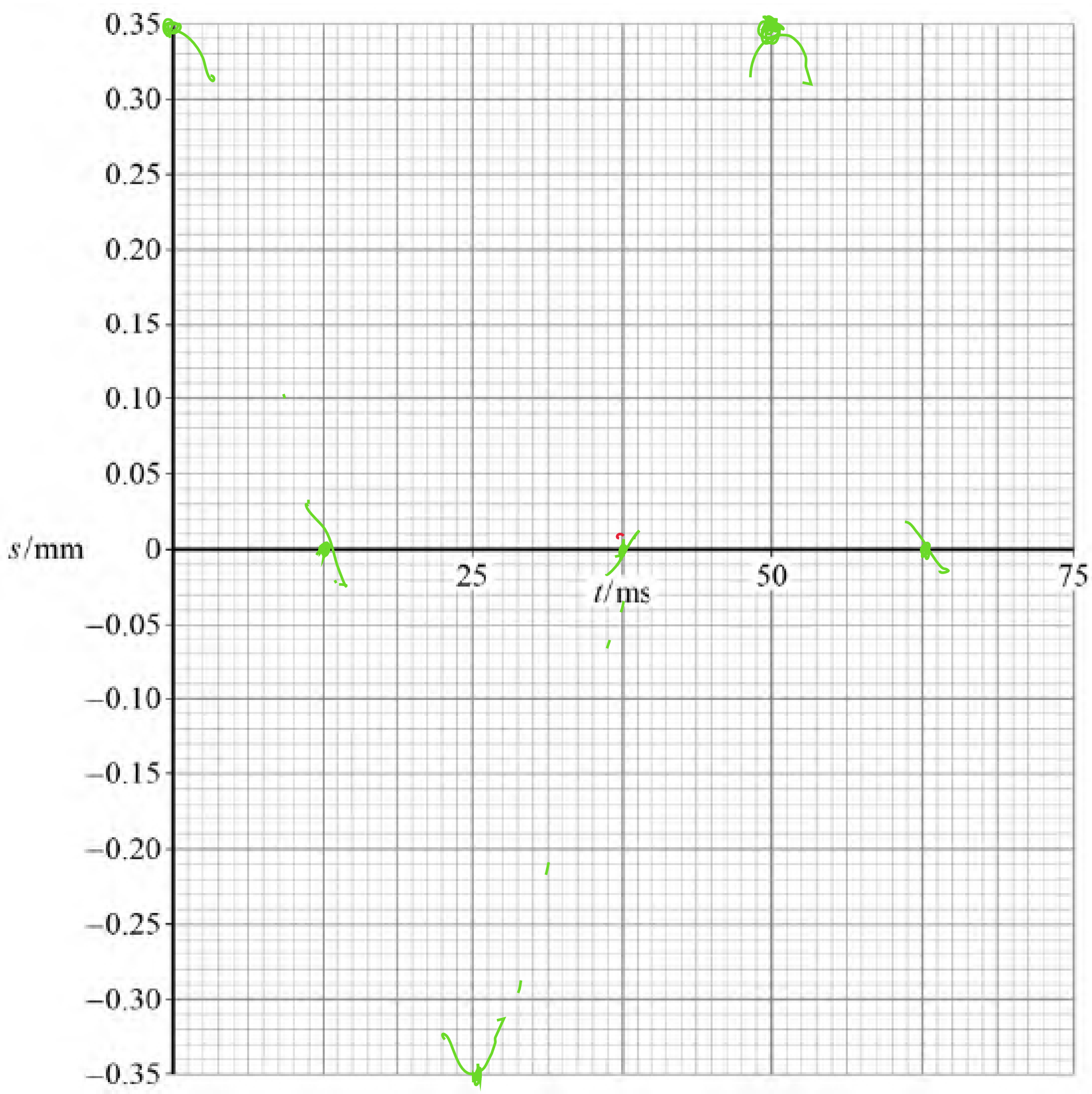
$\therefore \omega = 2\pi \times 20$

$= 40\pi$

(2)

(c) Draw on **Figure 3** the displacement−time (s−t) graph between 0 and 75 ms.

Figure 3



(1)

(d) State **one** time at which the plate has maximum potential energy.

When $v = 0$ is at

0, 25 ms etc

time = ______________________ s

(1)

(e) A small quantity of fine sand is placed onto the surface of the plate. Initially the sand grains stay in contact with the plate as it vibrates. The amplitude of the vibrating surface remains constant at 3.5×10^{-4} m over the full frequency range of the signal generator. Above a particular frequency the sand grains lose contact with the surface.

Explain how and why this happens.

depends on size of downward accel

from ~~top~~ point. If it is $< g$

then sand stays on plate

If $> g$ sand is "left behind"

and leaves the plate

(3)

(f) Calculate the lowest frequency at which the sand grains lose contact with the surface of the plate.

$a_{max} = \omega^2 A$ must be $= 9.8$ or more

$\sqrt{\dfrac{9.8}{3.5 \times 10^{-4}}} = \omega = 167.3 = 2\pi f \Rightarrow f$

26.6

frequency = ______________________ Hz

(2)

(Total 11 marks)

Mark schemes

1 **D**

[1]

2 C

[1]

3 (a) Radius of orbit = 5 tan 28.5 + 1 = 3.71 m ✓

1

Speed = 2 × 3.14 × 3.71/5.2 = 4.49 () ✓

For second mark only allow

Use of sin 28.5 gives orbit radius 3.39 m and speed = 4.1 m s^{-1}

Or

Forgets to add 1 giving radius 2.71 and speed 3.27 m s^{-1}

1

(b) Centripetal force = 85 × 4.49^2/3.71 = 460 N ✓

470 N if using 4.5 m s^{-1} leads to 1000 N

1

Centripetal force = T sin 28.5 ✓

Allow the following as ecf:

Forgetting to add the 1 m (using r = 2.71 m) leads to centripetal force = 630 N T = 1300 N)

*Using r =3.39 m as ecf from part **(e)** which leads to*

Centripetal force = 510 N giving T = 1070 N

1

T = 950 - 970 N ✓

1

OR

Weight = 85 × 9.8(1) 0r 834 N seen ✓

Weight = T cos 28.5 ✓

T = 950 (949) (N) ✓

OR

Centripetal force = 85 × 4.5^2/3.71 = 464 N ✓

Weight = 834 N ✓

$T = \sqrt{464^2 + 834^2}$ = 950 – 970 N ✓

Allow ecf for incorrect weight or centripetal force

Allow the following as ecf:

Forgetting to add the 1 m (using r = 2.71 m) leads to

Centripetal force = 630 N, T = 1050 N

Using r = 3.39 m leads to

Centripetal force = 510 N giving T = 980N

(c) Vertical (compressive) force **on the pole** increases ✓

1

Increases mass <u>increases weight</u> and hence tension in the rope(for the same angle) $_1$✓

1

<u>Centripetal Force</u>

on the acrobats/masses would be different/not equal

OR

Would be greater <u>on the more massive acrobat</u>(travelling at the same speed/same angle to vertical) $_1$✓

Unbalanced (horizontal) forces/resultant force exists (on the pole) ✓

OR

Unbalanced moments acting (on pole)/resultant torque acting (on pole) ✓

Causing the pole to sway/bend/move/ or tilt/topple the platform <u>toward more massive acrobat</u> $_1$✓

1
Max 3

[8]

4 B

[1]

5 **D**

[1]

6 (a) (use of $v = 2\pi f\sqrt{a^2 - x^2}$)

$v_{max} = 2\pi \times 2.0 \times 2.5 \times 10^{-2}$

$v_{max} = 0.314$ m s^{-1} ✓

(use of $E_k = ½mv^2$)

$54 \times 10^{-3} = ½m \times (0.314)^2$

$m = 1.1$ (kg) ✓

$f = \dfrac{1}{2\pi}\sqrt{\dfrac{k}{m}}$

$2.0 \times 2\pi = \sqrt{(k/1.1)}$ ✓

($k = (4\pi)^2 \times 1.1$)

$k = 173$ (172.8) ✓(N m^{-1})

Can

OR

$5.4 \times 10^{-3} = ½\, k\, (2.5 \times 10^{-2})^2$ ✓

$k = 173$ (172.8) N m^{-1} ✓

If either of these methods used can then find mass from frequency formula or from kinetic energy

OR

$54 \times 10^{-3} = ½\, F \times 2.5 \times 10^{-2}$

$F = 4.32$

$4.32 = k \times 2.5 \times 10^{-2}$

$k = 173$ (N m^{-1})

Accept 170 and 172.8 to 174

1
1
1
1

(b) (use of $f = \dfrac{1}{2\pi}\sqrt{\dfrac{k}{m}}$))

same mass so $f \propto \sqrt{k}$

thus frequency $= 2.0 \times \sqrt{3}$

frequency = 3.5 (3.46) (Hz) ✓

Allow CE from (a) for k or m

1
1

(c) Two from:

(resonance) peak / maximum amplitude is at a higher frequency ✓
due to higher spring constant ✓

(resonant) peak would be broader ✓
due to damping ✓

amplitude would be lower (at all frequencies) ✓
due to energy losses from the system ✓

First mark in each case for effect

Second mark for reason

2 marks max for effects

2 marks max for reason

Cannot award from sketch graph unless explained

First mark in each pair stand alone

Second mark conditional on first in each pair

1
1
1
1

[10]

7 (a) SHM is when

The acceleration is proportional to the displacement ✓

the acceleration is in opposite direction to displacement✓

2

(b) f = 1/T =1/0.05=20 Hz ✓

($(v_{max} = 2\pi f A)$

$A = \dfrac{0.044}{2\pi \times 20}$ ✓ (=3.5 × 10^{-4} m)

2

(c) Cosine shape drawn, maximum at *t*= 0, amplitude 3.5 × 10^{-4} m ✓

1

(d) (any of the following when the velocity is zero) 0.00s, 0.025s,0.050s or 0.075s ✓

1

(e) when the vibrating surface accelerates <u>down</u> with an acceleration less than the acceleration of free fall the sand stays in contact. ✓

above a particular frequency, the acceleration is greater than *g* ✓

there is no contact force on the sand ***OR***

sand no longer in contact when downwards acceleration of plate is greater than acceleration of sand due to gravity ✓

3

(f) (when the surface acceleration is the same as free fall)

$g = r\,\varpi^2 = A\,(2\,\pi f)^2$ ✓

$f = \sqrt{(g / A4\,\pi^2)} = (9.81/(3.5 \times 10^{-4} \times 4\,\pi^2))^{1/2} = 26.6(7)$ Hz ✓

2

[11]