

1

What is the angular speed of a car wheel of diameter 0.400 m when the speed of the car is 108 km h⁻¹?

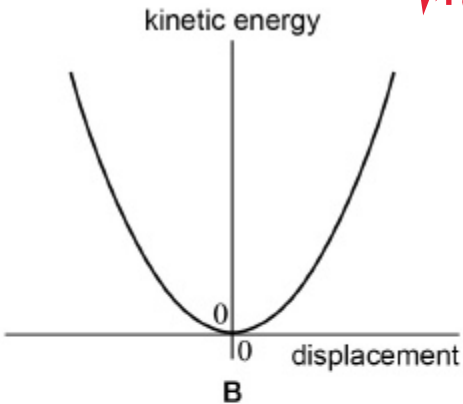
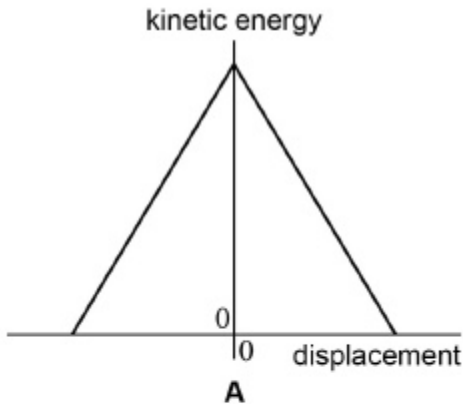
- A 75 rad s⁻¹
- B 150 rad s⁻¹**
- C 270 rad s⁻¹
- D 540 rad s⁻¹

Circum = $2\pi \times 0.4 = 1.27\text{m}$
108000m/h = 30m/s
 \therefore need $\frac{30}{1.27}$ turns/sec = 23.6
 $\therefore 148\text{ rad s}^{-1}$

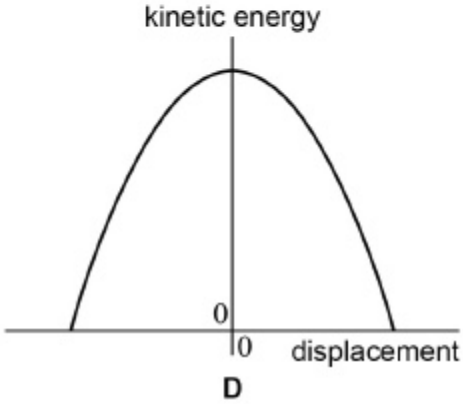
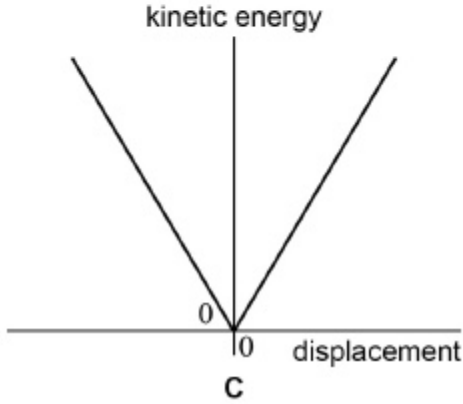
(Total 1 mark)

2

Which graph best shows how the kinetic energy of a simple pendulum varies with displacement from the equilibrium position?



max at d=0

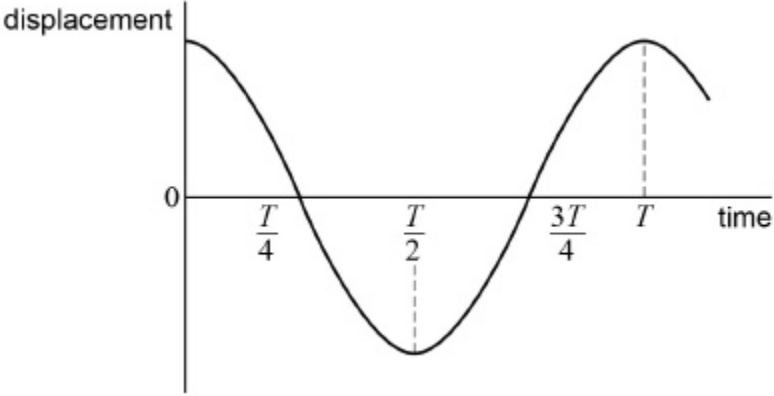


- A
- B
- C
- D**

(Total 1 mark)

3

The graph shows how the displacement of a particle performing simple harmonic motion varies with time.



Which statement is **not** correct?

- A** The speed of the particle is a maximum at time $\frac{T}{4}$
- B** The potential energy of the particle is zero at time $\frac{3T}{4}$
- C** The acceleration of the particle is a maximum at time $\frac{T}{2}$
- D** The restoring force acting on the particle is zero at time T

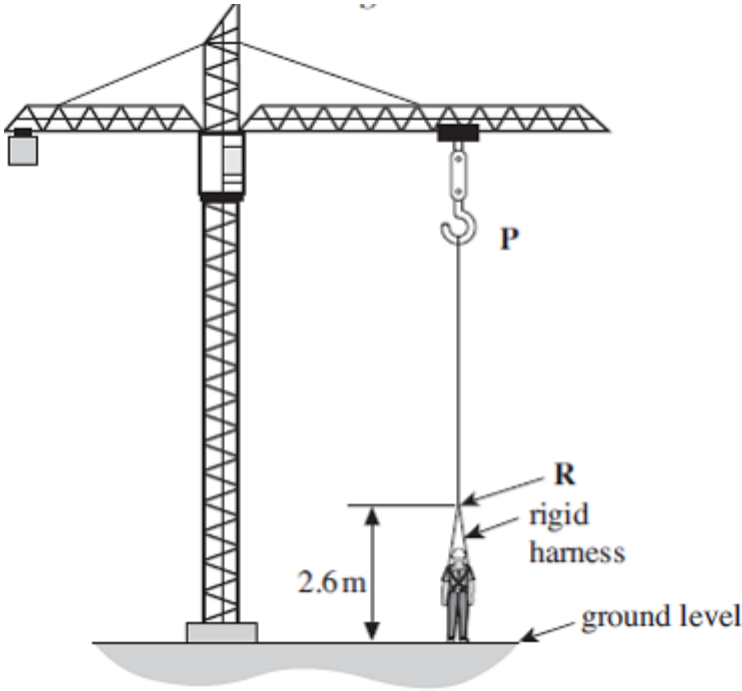
(Total 1 mark)

4

In a reverse bungee experience a 'rider' is catapulted high into the air. A designer creates a less extreme version for more timid participants, as shown in the figure below.

The rider is strapped into a rigid harness attached to one end of an elastic rope **PR**. The rider and the rope behave in the same way as a mass-spring system.

The rider is initially held at rest at ground level. The top end of the rope, **P**, is raised to stretch the rope. The rider is then released and moves upwards, reaching a maximum height when the rope is at its unstretched (natural) length. The rider then oscillates vertically until eventually coming to rest, suspended above the ground.



The rope has an unstretched length of 20 m. When stretched, the rope obeys Hooke's law and has a stiffness of 92 Nm^{-1} . In the following questions ignore the mass of the rope.

- (a) (i) The rider and harness have a total mass of 55 kg. Calculate the overall length of the rope when the rider comes to rest, suspended above the ground, at the end of the ride.

$$F = kx \quad \therefore x = \frac{F}{k} = \frac{55 \times 9.81}{92} = 5.86 \text{ m}$$

+ 20

overall length 25.9 m

(3)

- (ii) At the start of the ride, the lower end of the rope **R** is attached to the rigid harness at a point which is 2.6 m above the ground.

The top end of the rope, **P**, has to be adjusted so that the rope just becomes unstretched when the rider is at the highest point of the ride.

Determine the height of **P** above the ground.

Neglect air resistance in this part of the question.

$$\begin{array}{c} \underline{\quad 20 \quad + \quad 2x \quad + \quad 2.6 \quad} \\ \uparrow \qquad \qquad \qquad \uparrow \\ \text{orig} \qquad \qquad \text{amp } 5.9 \\ \text{length} \end{array}$$

height of point P 34.4 m

(1)

- (b) (i) Show that the frequency of oscillation of the rider on the end of the rope is about 0.2 Hz.

$$T = 2\pi \sqrt{\frac{m}{k}} \qquad T = 2\pi \sqrt{\frac{55}{92}}$$

$$f = \frac{1}{T} = \frac{1}{4.86} = 0.2 \text{ Hz}$$

(3)

- (ii) Calculate the maximum speed reached by the rider when the amplitude of the oscillation is 4.2 m.

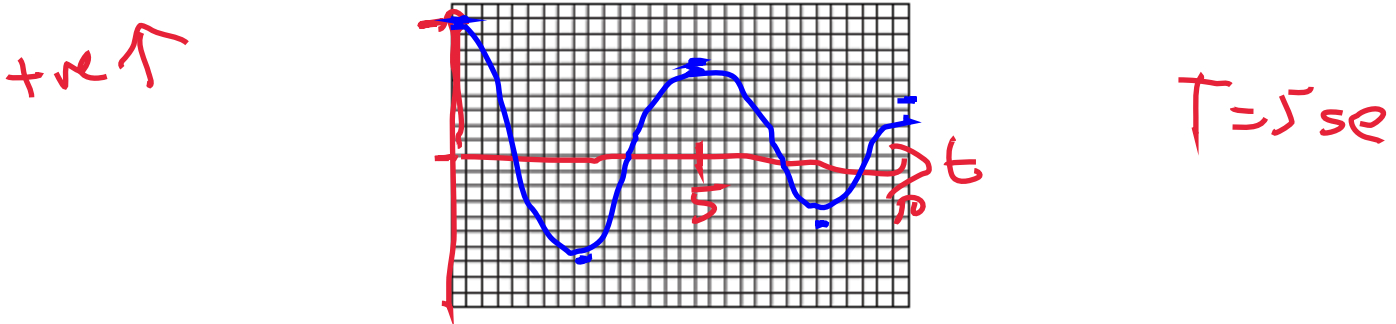
$$v_{\text{max}} = \omega A \qquad \omega = 2\pi f$$

maximum speed 5.43 ms⁻¹

(2)

- (iii) In practice, air resistance has an effect. Sketch below, a graph showing how you would expect the velocity to vary with time over the first two complete oscillations, from the instant the rider was released from ground level. Take an upward velocity as being positive.

Label the time axis with a suitable scale. No scale is required on the velocity axis.



(3)

- (c) (i) A rider of greater mass now uses the ride. Explain how the height of **P** has to be changed to produce the same initial amplitude of oscillations as that for the previous rider.

has to be varied
- amplitude ↑ because
initial stretch up

(3)

- (ii) A safety officer examines the design of the ride and thinks that, if the end **P** of the rope is raised too high so that the rope is stretched too much at the start, there is a risk that the rider could hit the ground after the first oscillation and suffer an injury. Describe what would happen to the rider during the ride in this case and explain why, even if air resistance is negligible, the safety officer's concerns are unfounded.

No risk. If no loss due to
air resistance it will only
return to the start point &
no further.

Possible at top the rope became
slack so free flight

To hit the ground you'd
need to break conservation
of energy

(3)

(Total 18 marks)

5

To celebrate the Millennium in the year 2000, a footbridge was constructed across the River Thames in London. After the bridge was opened to the public it was discovered that the structure could easily be set into oscillation when large numbers of pedestrians were walking across it.

- (a) What name is given to this kind of physical phenomenon, when caused by a periodic driving force?

resonance

(1)

- (b) Under what condition would this phenomenon become particularly hazardous? Explain your answer.

driving frequency is provided by people walking, if this is equal to the natural frequency of the bridge then large oscillations can build up risking structural damage

- (c) Suggest **two** measures which engineers might adopt in order to reduce the size of the oscillations of a bridge

measure 1 stiffen the structure to reduce vibration amplitude

measure 2 add dampers to absorb energy of walkers

(2)

(Total 7 marks)

or better, design the bridge more carefully so its natural freq \neq walking

Mark schemes

1	B		[1]
2	D		[1]
3	D		[1]
4	(a) (i)	use of $mg = k\Delta L$ or $F = k\Delta L$ and $F = mg$	
		extension = 5.9 m	C1
		total length = 25.9 m (allow 20 + their extension)	A1
			B1
			3
	(ii)	20 + twice (5.9) amplitude + 2.6; 34.4 m; allow ecf from ai	
			B1
			1
	(b) (i)	$T = 2\pi\sqrt{m/k}$ and $T = 1/f$ or $f = 1/2\pi\sqrt{k/m}$	
		correct substitution: allow for calculation of T (4.85 s)	B1
		0.21 or 0.206 (Hz)	B1
			3
	(ii)	substitutes data in $v_{\max} = 2\pi fA$	
		5.4 ms ⁻¹ (5.28 to 5.53)	C1
			A1
			2

- (iii) two complete oscillations shown with positive and negative velocities and acceptable shape (condone more than 2)

B1

and two from

period of 5 s used in graph (allow ecf for T from earlier part)

B1

start at 0 and positive velocity change at $T = 0$ with positive and negative velocities shown

B1

max velocity shown decreasing

B1

3

- (c) (i) it would have to raised

B1

rest extension would be greater/rider would be nearer the ground if extension unchanged

B1

the rider has to move down a distance = to the amplitude (5.9 m) from the new rest position

or with same initial extension/energy stored in rope, the rider would reach a lower height
amplitude would be lower

or due to the larger mass more energy ($= mgh$) is needed to reach the same height

so initial extension would have to be increased

B1

3

- (ii) the rope would become slack at the top of the ride so the rider would go into free flight/rider would overshoot the highest point

B1

the rider would fall and, with negligible air resistance, the rope would again absorb the energy arriving back at the start point **or** rider is more likely to fail to reach the ground after one oscillation due to energy losses/air resistance

B1

the PE gained (at the top of the flight) can (at most) only be converted back to the elastic energy that was stored in the rope at the start

(allow a statement to the effect that to hit the floor would contravene conservation of energy or require an energy input)

B1

3

[18]

5

- (a) forced vibrations or resonance (1)

1

- (b) reference to natural frequency (or frequencies) of structure (1)
 driving force is at same frequency as natural frequency of structure (1)
 resonance (1)
 large amplitude vibrations produced or large energy transfer to structure(1)
 could cause damage to structure [or bridge to fail] (1)

max 4

- (c) stiffen the structure (by reinforcement) (1)
 install dampers or shock absorbers (1)
 [or other acceptable measure e.g. redesign to change natural frequency
 or increase mass of bridge or restrict number of pedestrians]

2

[7]