

0 2

The Global Positioning System (GPS) uses satellites to support navigation on Earth.

0 2 . 1

One GPS satellite is in a circular orbit at a height h above the surface of the Earth. The Earth has mass M and radius R .

Show that the angular speed ω of the satellite is given by

$$\omega = \sqrt{\frac{GM}{(R+h)^3}}$$

$$F = \frac{GMm}{r^3} = m\omega^2 r$$

[2 marks]

$$\Rightarrow \frac{GM}{r^3} = \omega^2 \quad r = R+h$$

$$\Rightarrow \omega = \sqrt{\frac{GM}{(R+h)^3}}$$

0 2 . 2

Calculate the orbital period of the satellite when h equals 2.02×10^7 m.

[2 marks]

$$\omega = \frac{2\pi}{T} = \sqrt{\frac{GM}{(R+h)^3}} \quad \Rightarrow \quad 2\pi \sqrt{\frac{(R+h)^3}{GM}} = T$$

$$R+h = 2.657 \times 10^7 \text{ m}$$

$$T = 4.3 \times 10^4 \text{ s}$$

orbital period = _____ s

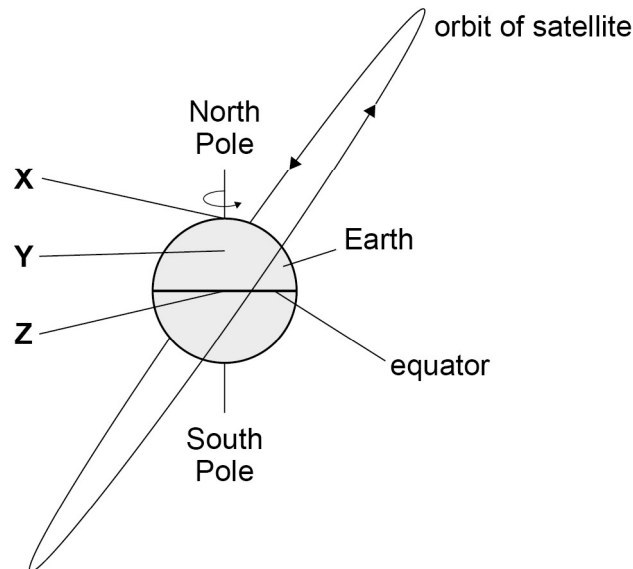


0 2 . 3

Figure 3 shows the orbital plane of the satellite inclined at an angle to the equator. **X**, **Y** and **Z** are locations on the Earth.

X is at the North Pole, **Y** is on a high mountain and **Z** is on the equator.

Figure 3



The satellite is to be launched from one of the locations.

State and explain which launch site **X**, **Y** or **Z** minimises the amount of fuel required to send the satellite into its orbit.

[2 marks]

Nearer to the equator the satellite has more initial E_k from the rotation of the earth. So Z is best

(Y might be better but the mountain would have to be high enough that the difference in potential is greater than the reduced E_k - but then hard to launch from a mountain)

Question 2 continues on the next page

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0 2 . 4 The satellite has a mass of 1630 kg.

Calculate the gravitational potential energy of the satellite when in the orbit in Question **02.2**.

[2 marks]

$$R_{th} = 2.657 \times 10^7 \text{ m} : \quad GPE = - \frac{GMm}{r}$$

gravitational potential energy = $2.44 \times 10^{10} \text{ J}$

0 2 . 5 A different satellite is in a higher circular orbit.

Explain how the linear speed of this satellite compares with the linear speed of the satellite in Question **02.1**.

[2 marks]

$$v = \omega r \rightarrow \omega = \frac{v}{r}$$

$$F = \frac{GMm}{r^2} = m\omega^2 r$$

$$\frac{GM}{r^3} = \frac{v^2}{r^2} \rightarrow \frac{GM}{r} = v^2$$

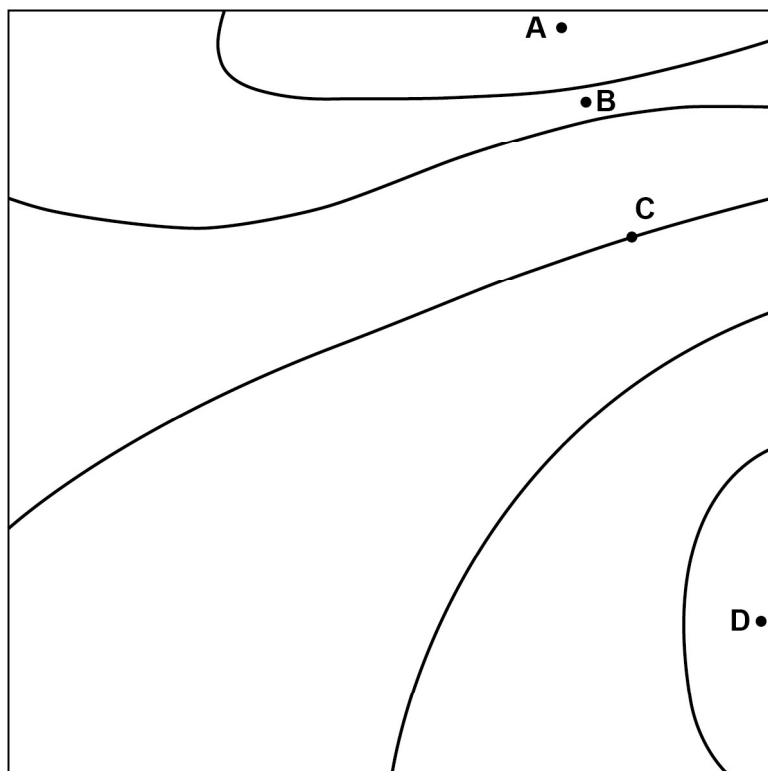
$$\sqrt{\frac{GM}{r}} = v$$

if r is greater then v will be smaller.
v and root r are inversely
proportional



1 1

The diagram shows gravitational equipotentials. Adjacent equipotentials are separated by an equal gravitational potential difference V .



Which point has the greatest gravitational field strength?

[1 mark]

A

B

C

D

gradient of potential is field strength



1 2 A planet has radius R and density ρ . The gravitational field strength at the surface is g .

What is the gravitational field strength at the surface of a planet of radius $2R$ and density 2ρ ?

- A $2g$
- B $4g$
- C $8g$
- D $16g$

[1 mark]

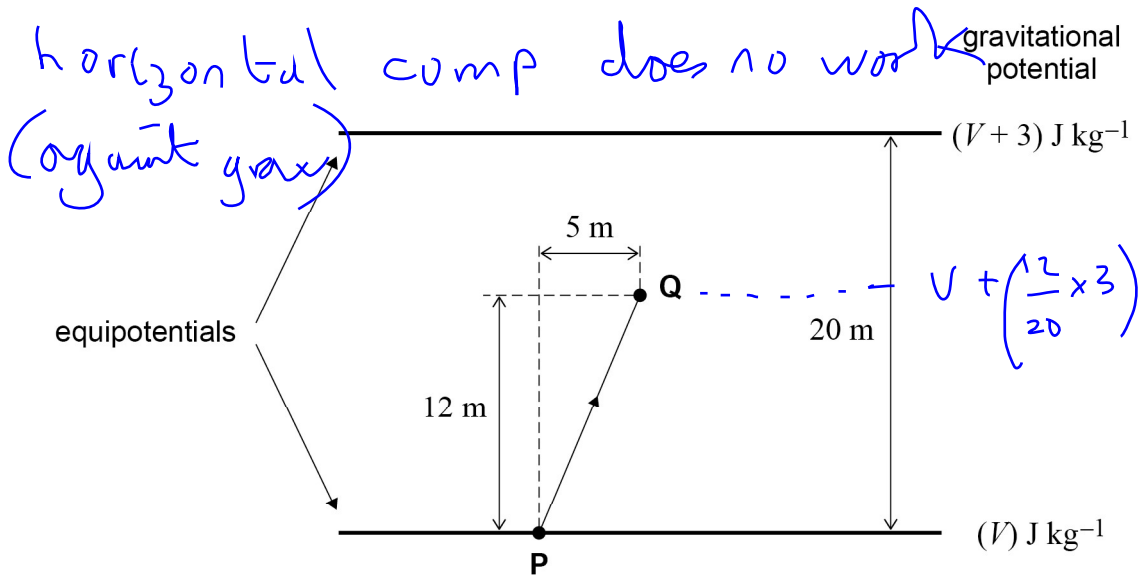
$$\rho = \frac{M}{V} \text{ vol} \therefore M = \rho V$$

$$M = \rho \frac{4}{3} \pi R^3$$

$$g = G \frac{M}{R^2} = G \frac{\rho \frac{4}{3} \pi R^3}{R^2}$$

$g \propto \rho R$
 $\therefore 4g$

1 3 The diagram shows equipotential lines for a uniform gravitational field. The lines are separated by 20 m.



An object of mass 4 kg is moved from **P** to **Q**.

What is the work done against gravity to move the object?

- A 7.2 J
- B 7.8 J
- C 10.2 J
- D 36 J

[1 mark]

$$\Delta W = m \times \Delta V$$

$$7.2 = 4 \times \frac{12 \times 3}{20}$$

Turn over ►

