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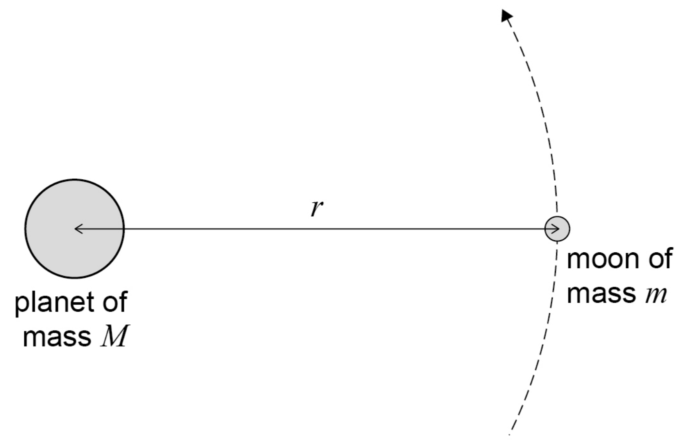
0 2 **Figure 2** shows a moon of mass m in a circular orbit of radius r around a planet of mass M, where $m << M$.

Figure 2



The moon has an orbital period T.
T is related to r by

$$T^2 = kr^3$$

where k is a constant for this planet.

0 2 . 1 Show that $k = \dfrac{4\pi^2}{GM}$

[3 marks]

$$\frac{GMm}{r^2} = \frac{mv^2}{r} \text{ and } v = \frac{2\pi r}{T}$$

$$\frac{GM}{r} = v^2 = \frac{4\pi^2 r^2}{T^2}$$

$$\frac{GM}{r^3} = \frac{4\pi^2}{T^2} \rightarrow \frac{r^3}{GM} = \frac{T^2}{4\pi^2}$$

$$\frac{4\pi^2}{GM} r^3 = T^2$$

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Table 2 gives data for two of the moons of the planet Uranus.

Table 2

Name	***T* / days**	***r* / m**
Miranda	1.41	1.29×10^8
Umbriel	4.14	**X**

0 2 . 2 Calculate the orbital radius **X** of Umbriel.

[2 marks]

$$T^2 = kr^3 \quad c \quad \left(\frac{T^2}{r^3}\right)_{\text{Miranda}} = \left(\frac{T^2}{r^3}\right)_{\text{Umbriel}}$$

$$\frac{T_m^2}{r_m^3} = \frac{T_u^2}{X_u^3} \Rightarrow \frac{r_m^3\, T_u^2}{T_m^2} = (r_u)^2$$

orbital radius = 2.65×10^8 m

0 2 . 3 Calculate the mass of Uranus.

[3 marks]

$$\frac{4\pi^2}{GM} r^3 = T^2 \Rightarrow \frac{4\pi^2 r^3}{G T^2} = M$$

best to use Miranda data at this point.

$$\frac{4\pi^2 \times (1.29 \times 10^8)^3}{6.67 \times 10^{-11} \times (1.41 \times 24 \times 60 \times 60)^2}$$

mass = 8.55×10^{25} kg

Question 2 continues on the next page

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Table 3 gives data for three more moons of Uranus.

Table 3

Name	Mass / kg	Diameter / m
Ariel	1.27×10^{21}	1.16×10^{6}
Oberon	3.03×10^{21}	1.52×10^{6}
Titania	3.49×10^{21}	1.58×10^{6}

$\sqrt{m/r}$

3.3×10^7

4.5×10^7

4.7×10^7

0 2 . 4 Deduce which moon in **Table 3** has the greatest escape velocity for an object on its surface.
Assume the effect of Uranus is negligible.

[3 marks]

$$\frac{GMm}{r} = \frac{1}{2}mv^2 \qquad \sqrt{\frac{2GM}{r}} = v_{esc}$$

$$v_{esc} \propto \sqrt{\frac{m}{r}} \quad \text{see table} \uparrow$$

Titania

note I used d instead of r - but this is ok because they are proportional to each other

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0 2 . 5 A spring mechanism can project an object vertically to a maximum height of 1.0 m from the surface of the Earth.

Determine whether the same mechanism could project the same object vertically to a maximum height greater than 100 m when placed on the surface of Ariel.

[3 marks]

Name	Mass / kg	Diameter / m
Ariel	1.27×10^{21}	1.16×10^{6}

if on earth we say its a unit test mass of 1Kg then the spring was storing mgh of Ep = 1x1x9.81 Joules

On Arial the Vg at the surface is GM/r - find g at the surface

Then use mgh to get new h. (This is ok because over 100m or so the value of g isn't going to change much)

$g = \frac{GM}{(d/2)^2}$ ✓ $\Rightarrow g = 0.25$ N/Kg

So $9.8\,J = mgh$

$9.8 = 1 \times 0.25 \times h$

$\Rightarrow h = 39\,m$ So NO ✓

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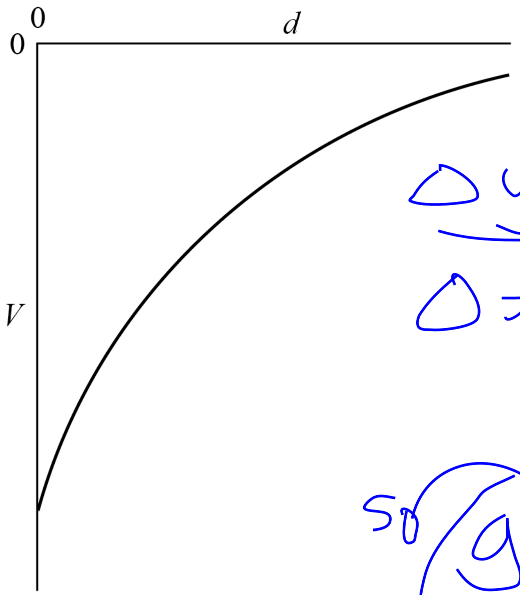
Turn over for the next question

1 2 The graph shows how the gravitational potential V varies with the vertical distance d from the surface of the Earth.



$\frac{\Delta y}{\Delta x} = -\frac{GM}{r}$

so $g = \frac{GM}{r^2}$

What does the gradient of the graph represent at the surface of the Earth?

[1 mark]

A potential energy

B mass of the Earth

C magnitude of the gravitational constant

D magnitude of the gravitational field strength ☒

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1 3 What is the angular speed of a satellite in a geostationary orbit around the Earth?

[1 mark]

A 1.2×10^{-5} rad s^{-1}

B 7.3×10^{-5} rad s^{-1}

C 4.4×10^{-3} rad s^{-1}

D 2.6×10^{-1} rad s^{-1}

$\omega = \frac{2\pi}{T}$

$T = 24\,hr \Rightarrow \omega = \frac{2\pi}{24 \times 60^2}$

Turn over for the next question