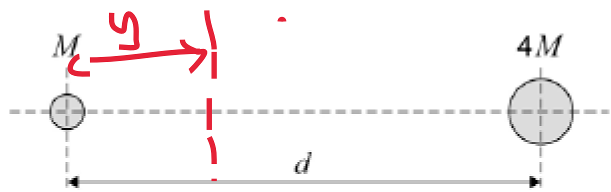
1 Two stars of mass M and $4M$ are at a distance d between their centres.

y

Click for Youtube walk through



The resultant gravitational field strength is zero along the line between their centres at a distance y from the centre of the star of mass M.

What is the value of the ratio $\frac{y}{d}$?

$$\frac{\cancel{G}\cancel{M}}{y^2} = \frac{\cancel{G}\,4\cancel{M}}{(d-y)^2}$$

$$\frac{1}{y^2} = \frac{4}{(d-y)^2} \Rightarrow 4y^2 = (d-y)^2$$

$$\Rightarrow 2y = d - y$$

$$\Rightarrow 3y = d \Rightarrow \frac{y}{d} = \frac{1}{3}$$

A $\frac{1}{2}$

B $\frac{1}{3}$

C $\frac{2}{3}$

D $\frac{3}{4}$

(Total 1 mark)

$T = 1$ day

2 A geosynchronous satellite is in a constant radius orbit around the Earth. The Earth has a mass of 6.0×10^{24} kg and a radius of 6.4×10^{6} m.

What is the height of the satellite above the Earth's surface?

$$r^3 = T^2 \frac{GM}{4\pi^2}$$

A 1.3×10^{7} m

B 3.6×10^{7} m

C 4.2×10^{7} m

D 4.8×10^{7} m

(Total 1 mark)

3 (a) State, in words, Newton's law of gravitation.

Force between 2 mass at distance r from centres is proportional to the multiple of the masses and inversely proportional to $\frac{1}{r^2}$. The constant is G

(3)

(b) By considering the centripetal force which acts on a planet in a circular orbit, show that $T^2 \propto R^3$, where T is the time taken for one orbit around the Sun and R is the radius of the orbit.

$F = \frac{\cancel{m}v^2}{r} = \frac{GM\cancel{m}}{r^2} \Rightarrow v^2 = GM/r \Rightarrow v = \sqrt{\frac{GM}{r}}$

$v = 2\pi r/T \Rightarrow T = \frac{2\pi r}{v}$

$T = \frac{2\pi r}{v} \Rightarrow T = \frac{2\pi r}{\sqrt{\frac{GM}{r}}} \Rightarrow T^2 = \frac{4\pi^2 r^2}{\left(GM/r\right)}$

$\Rightarrow T^2 = \frac{4\pi^2 r^3}{GM} \Rightarrow \underline{\underline{T^2 \propto r^3}}$

(3)

(c) The Earth's orbit is of mean radius 1.50×10^{11} m and the Earth's year is 365 days long.

(i) The mean radius of the orbit of Mercury is 5.79×10^{10} m. Calculate the length of Mercury's year.

$T^2 \propto r^3$ so for earth $\frac{T^2}{r^3} = k = 2.95 \times 10^{-14}$

$\frac{T^2}{r^3} = 2.95 \times 10^{-14}$ for Mercury too

$T^2 = 2.95 \times 10^{-14} \times (5.79 \times 10^{10})^3$

$\sqrt{5.7 \times 10^{13}}$ s

$= 7.6 \times 10^{6}$ s $\approx$ 87 day

(ii) Neptune orbits the Sun once every 165 Earth years.

Calculate the ratio $\frac{\text{distance from Sun to Neptune}}{\text{distance from Sun to Earth}}$.

$T^2 \propto r^3$ $\qquad \frac{T_E^2}{r_E^3} = \frac{T_N^2}{r_N^3}$

$\frac{r_N^3}{r_E^3} = \frac{T_N^2}{T_E^2} \Rightarrow \frac{r_N^3}{r_E^3} = \frac{(165)^2}{1}$

so $\frac{r_n}{r_E} = \sqrt[3]{165^2} = \frac{30}{1}$

(4)

(Total 10 marks)

4 The planet Venus may be considered to be a sphere of uniform density 5.24×10^3 kg m^{-3}. The gravitational field strength at the surface of Venus is 8.87 N kg^{-1}.

(a) (i) Show that the gravitational field strength g_s at the surface of a planet is related to the the density ρ and the radius R of the planet by the expression

$$g_s = \frac{4}{3}\pi G R \rho$$

where G is the gravitational constant.

$g = \frac{GM}{r^2}$

at R $g = g_s$ so $g_s = \frac{GM}{R^2}$ $\qquad \rho = \frac{M}{V} \Rightarrow \rho V = M$

so $g_s = \frac{G\rho V}{R^2}$ $\qquad V = \frac{4}{3}\pi R^3 \Rightarrow g_s = \frac{G\rho \, 4/3\pi R^3}{R^2}$

(2)

So $g_s = \frac{4\pi}{3} G\rho R$

(ii) Calculate the radius of Venus.

Give your answer to an appropriate number of significant figures.

$$\frac{3g}{4\pi G\rho} = R \qquad \frac{3 \times 8.87}{4\pi \times 6.67 \times 10^{-11} \times 5.24 \times 10^{3}}$$

$$= 6.06 \times 10^{6} \ (3sf)$$

radius = ____________________ m

(3)

(b) At a certain time, the positions of Earth and Venus are aligned so that the distance between them is a minimum.
Sketch a graph on the axes below to show how the magnitude of the gravitational field strength *g* varies with distance along the shortest straight line between their surfaces.
Consider only the contributions to the field produced by Earth and Venus.
Mark values on the vertical axis of your graph.

(3)
(Total 8 marks)

Mark schemes

1 B

[1]

2 **B**

[1]

3 (a) attractive **force** between point masses **(1)**
proportional to (product of) the masses **(1)**
inversely proportional to square of separation/distance apart **(1)**

3

(b) $m\omega^2 R = (-)\frac{GMm}{R^2}\left(\text{or} = \frac{mv^2}{R}\right)$ **(1)**

(use of $T = \frac{2\pi}{\omega}$ gives) $\frac{4\pi^2}{T^2} = \frac{GM}{R^3}$ **(1)**

G and M are constants, hence $T^2 \propto R^3$ **(1)**

3

(c) (i) (use of $T^2 \propto R^3$ gives) $\frac{365^2}{(1.50 \times 10^{11})^3} = \frac{T_m^2}{(5.79 \times 10^{10})^3}$ **(1)**

T_m = 87(.5) days **(1)**

(ii) $\frac{1^2}{(1.50 \times 10^{11})^3} = \frac{165^2}{R_N^3}$ **(1)** (gives $R_N = 4.52 \times 10^{12}$ m)

ratio = $\frac{4.51 \times 10^{12}}{1.50 \times 10^{11}}$ = 30(.1) **(1)**

4

[10]

4 (a) (i) $M = \frac{4}{3}\pi R^3 \rho$ ✓

combined with $g_s = \frac{GM}{R^2}$ (gives $g_s = \frac{4}{3}\pi G R \rho$) ✓

Do not allow r instead of R in final answer but condone in early stages of working.

Evidence of combination, eg cancelling R^2 required for second mark.

2

(ii) $R = \left(\dfrac{3g_z}{4\pi G\rho}\right) = \dfrac{3 \times 8.87}{4\pi 6.67 \times 10^{-11} \times 5.24 \times 10^{3}}$ ✓

gives R = 6.06 × 10^6 (m) ✓

answer to **3SF** ✓

SF mark is independent but may only be awarded after some working is presented.

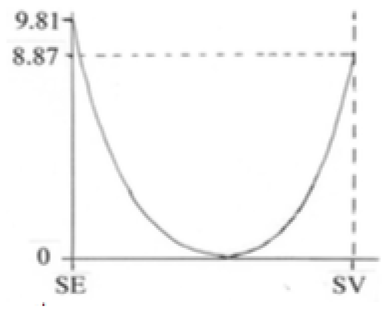
3

(b) line starts at 9.81 and ends at 8.87 ✓

correct shape curve which falls and rises ✓

falls to zeo value near centre of and to right of centre of distance scale ✓

[*Minimum of graph in 3rd point to be >0.5 and <0.75 SE-SV distance]*



For 3rd mark accept flatter curve than the above in central region.

3

[8]