5 This question is about the Boltzmann factor, $f = e^{-E/kT}$.

Fig. 5.1 shows how the Boltzmann factor varies with temperature for three processes: **A**, **B** and **C**.

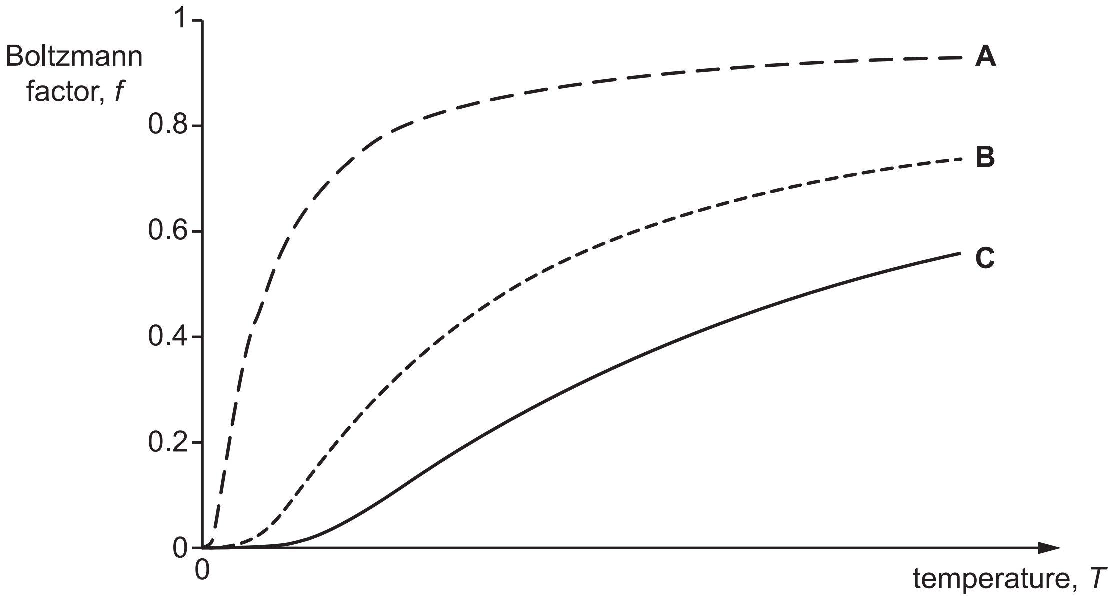

Fig. 5.1

(a) Explain how the graphs in Fig. 5.1 show that line **C** represents the process with the greatest activation energy *E*.

Boltzmann factor, f, is a measure of the proportion of particles with sufficient energy for the process to occur. E is the activation energy and KT is the average thermal energy. So bigger E means bigger E/kT means smaller f.

At a given temp the particles each have the same average thermal energy. So the line which has the smallest f at a given temperature has the highest activation energy.

[3]

(b) This part of the question is about the evaporation of liquids; the process in which molecules of the liquid gain sufficient energy to enter into the vapour.

(i) The Boltzmann factor for water molecules escaping the liquid and entering the vapour state is 4.9×10^{-8} at 310 K.

Calculate the activation energy required for a water molecule to escape into the vapour state at this temperature.

$T = 310$ $\quad f = 4.9 \times 10^{-8}$ $\quad f = e^{-\frac{E}{kT}}$

$\Rightarrow \ln(f) = -\frac{E}{kT}$ $\quad k = 1.38 \times 10^{-23}$ J/K

$\Rightarrow -kT \ln(f) = E$

activation energy = 7.2×10^{-20} J **[3]**

(ii) Explain how particles with an average energy lower than the activation energy gain enough energy to escape into the vapour.

Collisions are random and repeated. Some particles gain energy in a collision. If this happens enough then particles gain enough energy,

[2]

(iii)* The activation energy for a molecule of ethyl alcohol to escape into the vapour state is 6.6×10^{-20} J.

Calculate the Boltzmann factor at 310 K for this process and use ideas from the question to explain why a drop of ethyl alcohol feels colder on the skin than a drop of water.

$f = e^{-E/kT} \Rightarrow f = e^{-6.6\times10^{-20}/(310\times1.38\times10^{-23})}$

$\Rightarrow f = 2\times10^{-7}$ ~ alcohol

$f = 4.9\times10^{-8}$ for water

So BF alcohol is $\frac{2\times10^{-7}}{4.9\times10^{-8}}$ times higher = 4X

Rate of evap is faster with higher BF since more particles have the required energy.

More particle evaporating means quicker drop in temperature for the remaining liquid

[6]

$e^{-\frac{E}{kT}}$ E same at any T

20 At 300 K a process has an activation energy $\boxed{E = 10kT}$.

The temperature is raised to 330 K.

Which statement about the rate of the process is correct?

It will increase by

A 10% because temperature has increased by 10%.

B 10% because the mean square speed of the particles has increased by 10%.

C 9.1 times because $\frac{E}{kT} = \frac{3000k}{330k} = 9.1$.

D 2.5 times because $e^{\frac{-E}{kT}}$ has increased by $\frac{e^{-9.1}}{e^{-10}} = 2.5$ times.

Your answer [ ] **[1]**

300 $e^{-\frac{E}{k300}}$

$e^{-\frac{10k300}{k300}}$

330 $e^{-\frac{10k300}{330k}}$

$e^{-9.1}$

∴ Change is $\frac{e^{-9.1}}{e^{-10}}$

$\simeq 2.5\times$

6 This question is about conduction in metals and in semiconductors.

(a) A copper wire of length 1.5 m and radius 2.5×10^{-4} m has a resistance of 0.13 Ω at 20 °C. Calculate the conductivity of copper at this temperature.

$R = \rho \frac{L}{A} \Rightarrow \frac{RA}{L} = \rho$

$\frac{1}{\rho} = \sigma$ — (small Σ sigma)

$\rho = \frac{0.13 \times \pi (2.5 \times 10^{-4})^2}{1.5}$

$\rho = 1.7 \times 10^{-8}\ \Omega\,m$

$\therefore\ 5.9 \times 10^{7}\ S\,m^{-1}$

conductivity at 20 °C = 5.9×10^{7} S m^{-1} **[3]**

(b) A simple model of conduction suggests that each copper atom in the wire contributes one or more electrons to a cloud of free electrons that behave rather like particles in a gas. These electrons drift through the wire under the influence of an electric field.

The current I is given by the equation $I = nave$ where:

n is the number of free electrons in the material per m^3
a is the cross-sectional area of the wire
v is the drift velocity of the electrons
e is the electronic charge.

Calculate the drift velocity of the electrons when the copper wire in part **(a)** carries a current of 2.3 A. The number of free electrons per m^3 in copper = 8.5×10^{28} m^{-3}

$I = nave \Rightarrow v = \frac{I}{nae} = \frac{2.3}{8.5 \times 10^{+28} \times \pi (2.5 \times 10^{-4})^2 \times 1.6 \times 10^{-19}}$

$\frac{2.3}{2.7 \times 10^{3}}$

drift velocity = 8.6×10^{-4} m s^{-1} **[2]**

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(c)* The conductivity σ of semiconductors such as ntc thermistors increases dramatically with temperature T. The relationship is given by the equation

$$\sigma = C\, e^{-E/kT}$$

where C is a constant, k is the Boltzmann constant and E is the energy required to ionise an atom in the semiconductor.

Use the relationships given in the question to explain the effect of increasing temperature on the conductivity of metals and semiconductors, referring to the microscopic structure of the materials. No calculations are required. **[6]**

In conductors: temp up increases vibration of lattice ions, and the number of conduction electrons remains fixed. Each electron will therefore have a smaller mean free path and so the distance covered in unit time will decrease meaning I does not increase as much as the increase in potential would suggest

In semi conductors: again increasing the temperature increases the energy of the lattice atoms. As T up so the factor E/kT reduces meaning that e-(E/kT) will increase and so the conductivity will increase. This is because the number of conduction electrons has increased

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39 **(a)*** Boyle's Law states that at constant temperature, pressure is proportional to $\frac{1}{\text{volume}}$.

Explain how the random motion of particles gives rise to Boyle's law.

The particles have a range of speeds fitting a Maxwell-Boltzmann distribution.
When they collide they bounce of in every possible direction
In such collisions momentum is transfered
There will be a similar number of collisions per second per volume, meaning that the force/area is constant through out the gas, and on the container.

Given constant T then each collision will produce the same momentum change (and hence force). If we double the volume then there will be half as many particles per unit vol and therefore half as many collisions meaning half as much change in momentum and half as much force/area.

[6]

i.e. not mass so H_2 & Xe have same E_k

(b) Here are some data about trace gases in the atmosphere:

E_k depends on $\frac{3}{2}kT$

H_2 molar mass 2 grams
Xe molar mass 132 grams

(i) Calculate the ratio: $\dfrac{\text{speed of hydrogen molecule with average kinetic energy}}{\text{speed of xenon atom with average kinetic energy}}$.

Make your reasoning clear.

$$\cancel{\tfrac{1}{2}} m_{H_2} \overline{c^2} = \cancel{\tfrac{1}{2}} m_{Xe} \overline{c^2}$$

$$1 \cdot \overline{c_H^2} = 66 \overline{c_{xe}^2} \Rightarrow \frac{2}{66} = \frac{\overline{c_x^2}}{\overline{c_H^2}}$$

$$\Rightarrow \sqrt{\frac{1}{66}} = \frac{c_x}{c_H} \Rightarrow \frac{c_H}{c_x} = \sqrt{66}$$

ratio = ...8 : 1.. **[3]**

(ii) The escape velocity for planet Earth is 11.2 km s^{-1}.

Use the Boltzmann factor to estimate the number of H_2 molecules per mole with sufficient energy to escape the atmosphere and the Earth's gravitational field at a temperature of 288 K.

Molar mass $H_2 = 2g$

$\therefore m$ of $1\ H_2 = \frac{2}{6.02 \times 10^{23}} = 3.3 \times 10^{-27}$ kg

$$E_{escape} = \frac{1}{2} \times m v_{esc}^2$$

$$E_e = \tfrac{1}{2} \times 3.3 \times 10^{-27} \times (11.2 \times 10^3)^2 = 2.1 \times 10^{-19} J$$

No escaping $= N_A \times e^{-\frac{E}{kT}}$ out of 1 mole

$$\boxed{\frac{E}{kT} = 52}$$

$$6 \times 10^{23} \times e^{-2.1 \times 10^{-19} / (1.4 \times 10^{-23} \times 288)}$$

number = ...14.. mole^{-1} **[4]**

8 – 52 accepted

(BF calculations are very sensitive so rounding makes a big difference)

SECTION C

Answer **all** the questions.

36 This question considers some of the evidence for a Hot Big Bang start to our expanding universe.

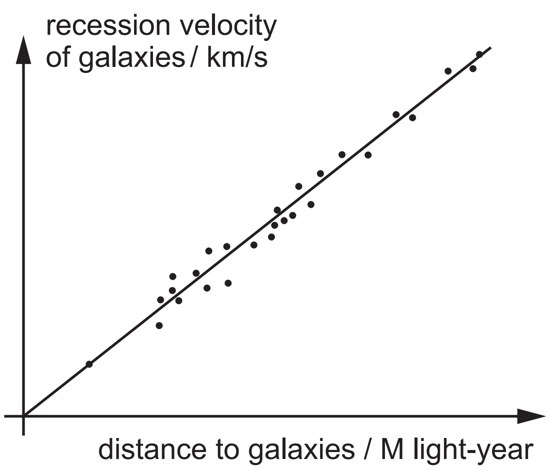

Fig. 36.1

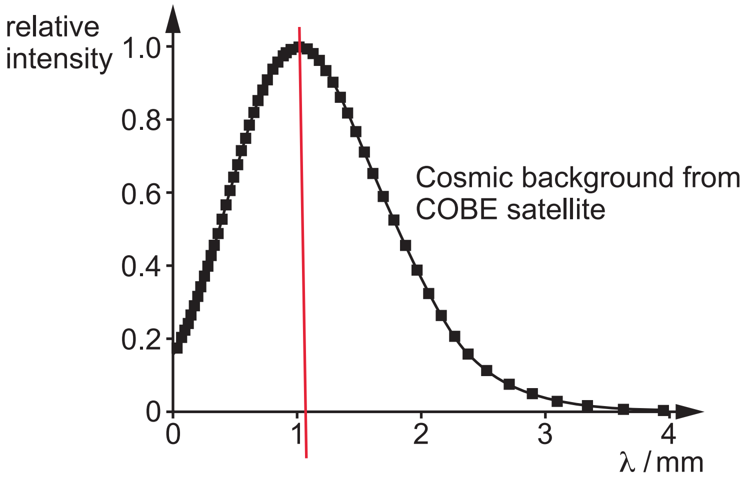

Fig. 36.2

(a) Explain how the graph(s) show evidence that the universe started from:

(i) a big bang expansion

36.1 shows distance and vel are proportional. This implies that if we were to 'rewind' then all the matter would arrive a point since the faster galaxies will have moved further than the slower ones.

.... **[2]**

(ii) a hot state.

This graph shows a peak temperature/energy in the microwave part of the e/m spectrum. If the galaxies are moving away then we would expect a red shift (stretching) of the waves, implying that in the past the energy was at a higher value.

.... **[2]**

(b) The intensity spectrum of thermal radiation depends on temperature T. Photons at the **peak** of intensity have energy $\varepsilon \approx 5kT$.

Use this approximation and data from **Fig. 36.2** to estimate the temperature of the cosmic microwave background radiation (CMBR).

peak λ at 1.1 mm. f is $\frac{c}{\lambda} = \frac{3\times10^{8}}{1.1\times10^{-3}} = 2.7\times10^{11}$ Hz

$\therefore \ \varepsilon = hf = 1.8\times10^{-22}$ J

$k = 1.4\times10^{-23}$

$\frac{\varepsilon}{5k} \simeq T \Rightarrow T = 2.6$ K (range $2.5 \to 3.2$ K)

temperature = ... K **[4]**