Circular Motion

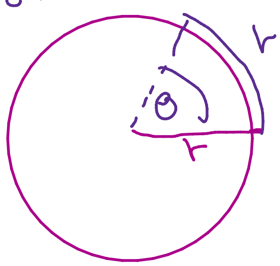
27 November 2019 10:20

Radians

A radian is another way of measuring angles. It is the angle created l "subtended") when an arc the length or a radius is made.

Circumference $= 2\pi r$

So the length r goes into the circle 2π times



One turn $= 360°$ ... & the length r goes into one turn 2π times. ← 6.28....

So.... that means if we do $\frac{360}{2\pi}$ we get how many degrees θ is.

$\frac{360}{2\pi} = 57.298$ (3 dp) We give 57.3° a name 1 radian or 'rad'.

So 1 rad = 57.3°. The radian is very useful ~ make sure your calculator is set to radians. We usually/often measure phase difference in radians.

You need to be comfortable thinking in terms like π rads or π/2 as 'half a cycle/wave' and 'quater cycle/wave'.

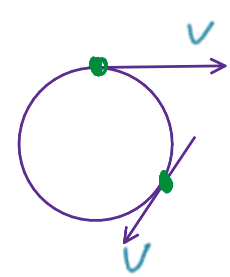
ω

Angular Velocity

This is just what it says it is - for something that is rotating, or undergoing periodic motion (think SHM) we can talk about the number of degrees, or more likely radians, that are turned in 1 second. This is called 'angular velocity' and is rads/sec or degrees/sec... And is given the symbol ω (capitol omega ...) not W or w.

$$\omega = \frac{\theta}{t}$$

Linear speed, sometimes called tangential velocity - this is a vector that you'd get at any point of the circle showing the 'velocity at that instant'. Ii is always a tangent to the circle. In the diagram I show this for two points for something turning clockwise.



This leads to the next equation

$$\omega = \frac{v}{r}$$

Frequency and Period

This is easy. If frequency is the number of waves in a second, then 1/f is the time take for one wave (in seconds). We call this the period. This deceptively easy equation comes up over and over again.

$$T = \frac{1}{f}$$

From this we can link angular velocity. For one complete turn the object has gone through 2π radians in a time T. Therefore the angular velocity is:

$$\omega = \frac{2\pi}{T} \qquad \omega = 2\pi f$$

Centripetal Acceleration

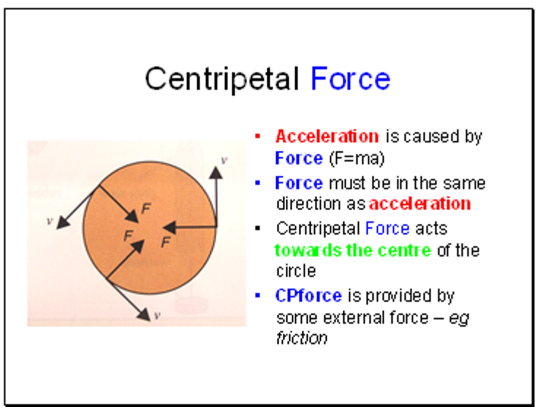
For something moving in a circle we know the velocity is always change even though the speed is constant (think vector). Since v is changing this means that there must be an acceleration. This is called centripetal acceleration and is towards the centre of the circle.

Since Newton 2 says that force is proportional to acceleration for a given mass there must be a force - it too is towards the centre. As always it is found with F = ma

$$a = \frac{v^2}{r} \text{ and } a = \omega^2 r$$

To get to the force you simply multiply the acceleration by mass.

$$F = \mathrm{m}\frac{v^2}{r} \text{ and } F = \mathrm{m}\omega^2 r$$

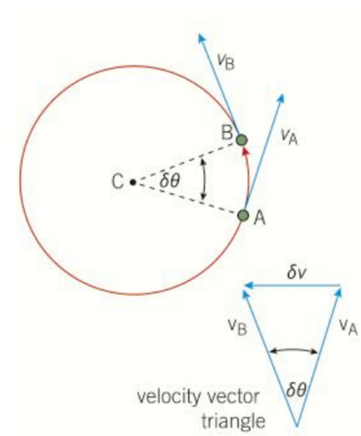


http://www.sliderbase.com/spitem-702-1.html

Note the last point. Something has to provide that centripetal force.
EG gravitational attraction,
Contact force (reaction).

Proof of Centripetal Accel Formula

object moving uniformly from A to B in circular path radius r



- Short time δt, $\delta\theta$ is small
- AB is $\delta S = v\delta t$ moving through $\delta\theta$
- If we make a vector triangle of V_A & V_B we can see the change in V, δV

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- $\delta V = V_B - V_B$
- The triangle ABC is similar to the vector triangle.

$\delta\theta$ must be small so arc $\approx$ straight

so $\frac{\delta V}{V} = \frac{\delta S}{r}$

- $\delta S = v\,\delta t$

so $\frac{\delta V}{V} = \frac{v\,\delta t}{r}$

so.... $\frac{\delta V}{\delta t} = \frac{v^2}{r}$ & $\frac{\delta V}{\delta t} = \text{accel}$