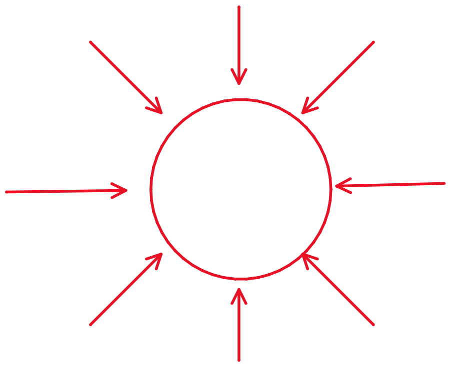
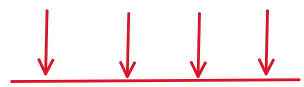
Gravitational Fields

04 December 2019 09:08

Field is an area where there is a NON CONTACT force. When you push something you have to touch it to effect it. Not so in a field. Its 'action at a distance'.





A radial field - lines of force:
- Point towards the centre of mass
- Are closer together where the force is strongest

Near the surface the field lines are approximately parallel, meaning the force is approx constant

Newton's Law of Gravitation

This can be written in a variety of ways - eg Mm or M_1M_2 - you have to be clear what your 'M's are.

$$F = \frac{GM_1\mathrm{M}_2}{r^2} \quad \cdots \quad 1$$

There is a common question which asks you to put Newton's Law of Gravitation into words. This means you write something like:
- The force between two mass is:
- Proportional to the product of the masses
- Inversely proportional to the square of the distance between the two center of masses
- Has G = 6.67x10-11 Nm2kg-2 as its constant of proportionality

Field Strength

Gravitational field strength is the 'force per unit mass'. This unit mass is nearly always 1Kg (and positive - well at the moment - assuming negative mass is not a thing waiting to be discovered....)

It is given the symbol g and you are already familiar with it from GCSE physics

$$g = \frac{F}{m} \cdots 2$$

note how similar this is to Newton Two F = ma) as you know this is approximately 9.81 Nkg-1 on the earth's surface. So every Kg of mass experiences a force of 9.81 N. It can also be called the 'acceleration due to gravity)

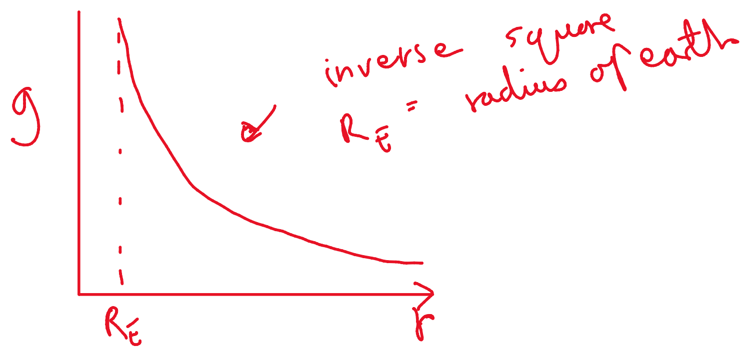
Little g, the gravitational field strength, is intimately related to Newton's Law of Gravitational attraction. You can substitute for F in 2 from 1 to get a formula that works in radial fields.

You can get the same thing by if you take $F = \frac{GM_1\mathrm{M}_2}{r^2}$ and set M2 = 1Kg .... it becomes $F = \frac{GM_1}{r^2}$ and we don't bother to write the 1 for M2. So we end up with

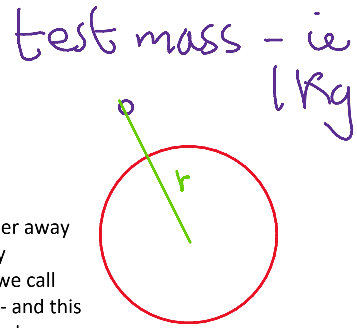
$$F = \frac{GM}{r^2} \cdots 3$$

Remember there ***are*** 2 masses here.... It's just that one is 1kg .... So the M is the mass of the planet that whose field the 1Kg is sitting in.

Both equations 1 and 3 are examples of 'inverse square laws' - a relationship that crops up a lot in physics. I might do a separate sheet on it.



Gravitational Potential (V)



This is about the energy that an object has when it sits in a gravitational field.

The small mass feels a force towards the center of the earth. If you wanted to move it out further away from the earth you would need to do work against that force of attraction. As you know, energy transferred is force times distance. Therefore, as you push the object out it is gaining energy - we call this gravitational potential energy. This would be easy to work out if g (the force) was constant - and this is what you did when you used mgh in gcse. However, we are growing into big physicists now and we need to know what to do when the value of g changes.... As it will do if we increase r

$$v = -\frac{GM}{r} \quad \ldots 4$$

There is a lot going on in this formula....

It is negative because you have to **do work to increase** r. If you are doing work then it is gaining energy.

As you go further out the attractive force decreases and so you need less energy to move the mass than when you were closer to the planet. But where will the mass have zero V? It was decided that zero V would be fixed at an infinite distance away (∞). We say the value of V at ∞ is zero - this makes sense - you are so far away from the source of the grav field that the force is effectively zero so work is done in moving the test mass.

When its closer however, as you increase r it is gaining energy, and the it is zero at infinity then it must be negative at r. Takes time to get your head round this.

This idea of potential being negative for attractive forces is going to crop up again in electric fields.

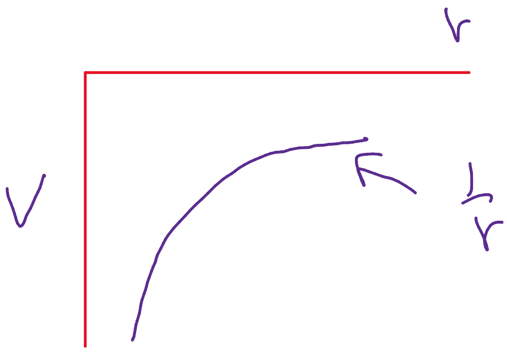
An example:

Suppose you have the sun with a mass 1.99×10^{30}Kg. What is the potential at $r = 1.5 \times 10^{11}$m?

We are going to find V = and putting the values into equation 4 above... Gives -885MJ/Kg

So this means that it would need 885MJ to move a 1kg object from 1.5×10^{11}m out to infinity.

If you had a rocket of mass 123Kg (yes a small rocket) at this distance from the sun then its Gravitational Potential energy would be -885MJ/kg x 123kg = -1088MJ meaning it would need -1088MJ to escape from the sun's grav field completely - ie to get to infinity.

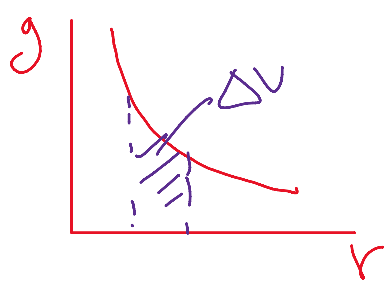


the gradient here is $\frac{\Delta V}{\Delta r}$. If you divide $V = -\frac{GM}{r}$ by r you get $V = \frac{\left(-\frac{GM}{r}\right)}{r}$

this is g!! $\Rightarrow$ $= \underline{-\frac{GM}{r^2}}$

The upshot of this then is that the gradient of a V graph gives you the g or in words...

Gradient of the gravitational potential gives you the gravitational field strength like this.



If you take g & multiply by r you get.... V (check) so area under curve equals ΔV