Pressure of an Ideal Gas - 1

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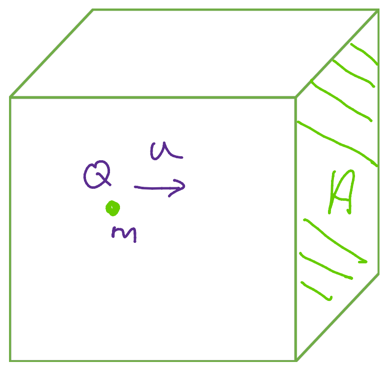
Grab your beverage of choice, turn off the music and follow closely!

We start with a cube....

← L → all sides are L



particle will collide with wall A

Positive to the right

① A particle, Q, of mass m is travelling to the right at velocity u

Recall momentum is mass x velocity

Q has momentum = mu

It hits the wall, bounces back along the same path and now its going at -u.

Change in momentum is final mom - initial mom:

= -mu-+mu = -2mu

② Assuming Q doesn't collide with any other particle (really?) then it hits the opposite wall & bounces back to hits A again.

$v = \frac{d}{t} \Rightarrow t = \frac{d}{v}$

Assuming Q doesn't collide with any other particles (really?) then it hits the opposite wall and rebounds back to hit A again.

If we time from a collision on A with the next collision on A we get:

$$t = \frac{2L}{u} \Rightarrow$$

If we know the time between collisions we can get the number of collisions per second

$$\Rightarrow \text{collisions per second} = \frac{u}{2L}$$

between collisions.

③ We are after something about the force on the wall. Rate of change of momentum is useful - so let's find that next

(I will use mom for momentum rather than P)

$$\Delta \text{mom} / \text{sec} = \Delta \text{m per collision} \times \text{no of collisions in 1 sec}$$

$$\text{ie: } -2mu \times \frac{u}{2L} = -\frac{mu^2}{L}$$

Rate of change of momentum is called Impact and is equal to the force on the wall.

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So Q exerts a force on the wall and by Newtons law's we can say that the force of the wall back on Q =

$$F = -\frac{mu^2}{L}$$

the proof's flits around here. So in the next they line talk about the force of the molecule on the wall. Not difficult, just confusing.

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Now its time to generalise - well, take the first step in going from one particle to a gas. (albeit an ideal gas...)

Particle Q is one of many particles. Each ahs the same mass. They will be going at various different velocities, but let's say they are always towards A (ie only going in the x direction). These velocities will naturally give each particle a different rate of change of momentum in their collisions with A, and again they don't hit each other!!!!

The values of these velocities will be u_1, u_2, u_3 etc. The total force on A will then just be the sum of all these 'rate of changes of momentum'.

$$F = \frac{mu_1^2}{L} + \frac{mu_2^2}{L} + \frac{mu_3^2}{L} + \dots$$

$$\Rightarrow F = \frac{m}{L}\left(u_1^2 + u_2^2 + u_3^2 + \dots\right)$$

6

Clearly this sum would take rather a long time to do (like forever, and that's assuming you could measure the various values of u)

Thankfully there is a really useful definition called the 'mean square speed'. This is exactly what it says.... Get all speeds, square them individually, add all the squares up and then divide by the number of speeds to get the mean. This is written with a 'bar' over the top - be sure to put your bar over the U and the square like this:

$$\overline{u^2} = \frac{u_1^2 + u_2^2 + u_3^2 + \dots}{N}$$

N = number of particles

7

Here's an important step.... Let's put this mean square speed into the force equation above:

$$F = \frac{m}{L}\left(u_1^2 + u_2^2 + u_3^2 + \dots\right) \text{ and } \overline{u^2} = \frac{u_1^2 + u_2^2 + \dots}{N}$$

giving $F = \frac{m}{L}\overline{u^2}$ BUT careful ....

This expression is now the force for 1 molecule since. The one in step 6 was for N molecules and so we have to multiply the version we have just got by N giving...

$$F = \frac{Nm\overline{u^2}}{l}$$

8 We now remember that P is force/area so our final eqn in 7 can easily be turned into pressure. What is the area of the side A ... It has length l so its area is l^2

$$P = \frac{F}{A} = \frac{Nm\overline{u^2}}{l \times l^2}$$

$l^2 \nearrow A$

$$l \times l^2 = V$$

$$\text{so} \quad P = \frac{Nm\overline{u^2}}{V}$$

this is so powerful. We have the velocity of a particle.... from which we can get the pressure of a gas! Cool - individual properties leading to bulk properties.

There is always more....

We made some assumptions. No collisions between particles and only moving in the x direction. The first we have to live with (remember what properties an ideal gas has....) the second we can do something about.

Clearly the particle is unlikely to be travelling along the x direction only. It will have components in both the x, y and z directions (ie travelling in 3d rather than 1d)

We call the actual speed of a particle c and the components in each dimension u, v and w for the x,y,z. From Pythagoras' theorem we can say:

$$c^2 = u^2 + v^2 + w^2$$

Treating each of the N particle the same (again, a lovely approximation!) we can there come up with a generalised mean square speed:

$$\overline{c^2} = \overline{u^2} + \overline{v^2} + \overline{w^2}$$

The particles are moving randomly. This leads to the next step where we treat all three of the mean square components as having the same size (remember that because we are squaring any -ve numbers become positive)

so $\overline{u^2} = \overline{v^2} = \overline{w^2}$ giving

$$\overline{c^2} = \overline{u^2} + \overline{u^2} + \overline{u^2} \quad .... \quad !!!$$

$$\text{so} \quad \overline{c^2} = 3\overline{u^2} \Rightarrow \overline{u^2} = \frac{\overline{c^2}}{3}$$

This is where the mysterious one third that keeps cropping up comes from.

Previously when thinking of just one dimension we had:

$$p = \frac{Nm\overline{u^2}}{V} \quad \text{or} \quad pV = Nm\overline{u^2}$$

Now sub in our new value for c - ie generalising to 3 dimensions

$$pV = Nm\frac{1}{3}\overline{c^2}$$ and tidying up

$$pV = \frac{1}{3}Nm\overline{c^2}$$