Pressure of an Idea Gas - 2

09 December 2019 10:51

If you are happy with part 1 then now we are just doing a bit housework. The previous formula has lots of implications and despite my trying to draw your attention to the assumptions proves to be very powerful indeed.

$$pV = \frac{1}{3} N m \overline{c^2}$$

Root Mean Squared (rms) is useful

You will come across this in the magnetic fields topic as well. RMS is a useful concept:

$$rms = \sqrt{\text{mean square speed}}$$

$$= \sqrt{\overline{c^2}} = c_{rms} \Rightarrow \overline{c^2} = c_{rms}^2$$

so we get:

$$\underline{pV} = \frac{1}{3} \underline{N m (c_{rms})^2}$$

As we reflect on these equations more important ideas emerge.... But first here's a list of assumptions regarding the ideal gas.

- Molecules move randomly
- Motions follows newton's laws
- Collisions are perfectly elastic
- Particles always move in straight lines (except when colliding)
- Any forces that act in a collision act for a very short time.

The last point suggests that there are no forces between the particles when they are not colliding. Therefore a particle has no potential energy.

Real gases approximate to ideal gases provided the pressure isn't too big (whatever that means) and the temperature is reasonably high cf their boing points -again what does 'reasonably' mean?

We have got a formula for pressure of a gas based on the average speed of a particle.

Towards Linking KE of a particle and absolute Temperature

We have:

$$pV = nRT \quad \text{and} \quad pV = \frac{1}{3} N m (c_{rms})^2$$

So $nRT = \frac{1}{3} N m (c_{rms})^2$

You can see that $\frac{1}{3} N m (c_{rms})^2$ is beginning to look like $\frac{1}{2} m v^2$.... KE.

We can "force" this by multiplying both side by $\frac{3}{2}$

$$\Rightarrow \frac{3}{2} nRT = \frac{3}{2} \times \frac{1}{3} N m (c_{rms})^2$$

$$\Rightarrow \frac{3}{2} nRT = \frac{1}{2} N m (c_{rms})^2$$

$\div N$

$$\Rightarrow \frac{3nRT}{2N} = \underbrace{\frac{1}{2} m (c_{rms})^2}_{\text{Looks} \rightarrow E_k}$$

So $E_k = \frac{3nRT}{2N}$ 1

Recall that $N = nN_A$ and that $\kappa = \frac{R}{N_A}$

Re-arranging both of these for N_A and then setting them equal gives:

$Nk = nR$ ---- 2

Sub for nR in 1 gives

$$E_K = \frac{3}{2}\frac{NkT}{N} \Rightarrow \underline{\underline{E_K = \frac{3}{2}kT}}$$

This is quite an expression - you have a figure for the average kinetic energy in a gas which DEPENDS ONLY on the temperature in kelvin (and a constant 3k/2)

Finally (yes!) we know that $k=R/N_A$ so subbing into the last formula for Boltzmann's constant gives an expression for E_k based on the molar gas constant.

$$E_K = \frac{3}{2}\frac{R}{N_A}T$$

You I know I said finally..... There is a whole bunch of stuff here about the scientific method. You know, like the 3 gas laws were developed from practical work (so are empirical) whereas the derivations of Kinetic Theory are exactly that - theoretical. But you can look this stuff up in your textbooks.