Radioactivity 003 Exponential Law of Decay

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Radioactive decay is a random process so you cannot say when an individual nuclei will decay. However, if you have a large number of nuclei then you can see clear patterns. So a sample of a particular radioactive isotope will decay at the same rate - the same proportion will decay in a given time.

The decay constant λ is the probability that a given nucleus will decay per second. It has units S^{-1}

Activity is the number of decays from a sample in a second - A - and is measured in Becquerels (Bq)

N is the number of nuclei in a sample

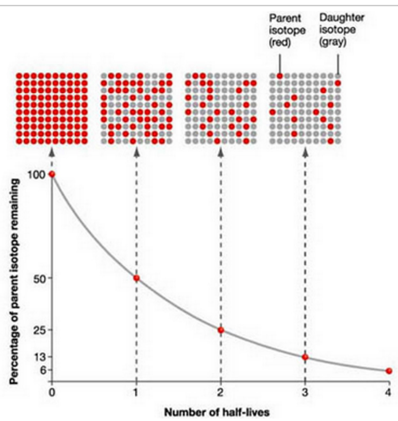
Therefore A = λN

Since activity is defined as the rate of change of N we can say:

$$\frac{\Delta N}{\Delta t} = -\lambda N$$

negative because N is decreasing as nuclei decay

Half Life

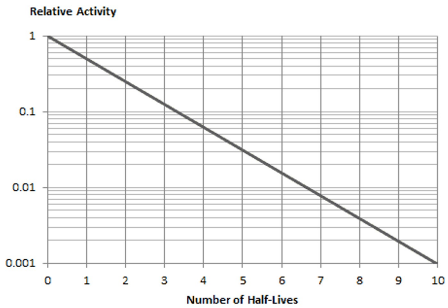


Time taken for the number of nuclei to half.

Graphs shows 4 consecutive half life periods[[[[[[[[[on the x axis - which is therefore a time axis, with the half life time written as '1', '2' etc

Note how the percentage of nuclei falls by 50% every half life time.

This is an exponential decay.



Shows an exponential decay with ativity up the y axis. The bottom graph is what happens if you take natural logs of the activity - it is the straight line so beloved of physicists.

Gradient is the negative of the decay constant

$N = N_0 e^{-\lambda t}$ or $A = A_0 e^{-\lambda t}$

N_0 = number at start of time interval t } ditto A_0

At $t_{1/2}$ number N will be $\frac{N_0}{2}$

So... $N = N_0 e^{-\lambda t}$ becomes

$\frac{N}{N_0} = e^{-\lambda t_{1/2}} \Rightarrow \frac{1}{2} = e^{-\lambda t_{1/2}}$

Take natural logs (ln)

so $\ln\left(\frac{1}{2}\right) = -\lambda t_{1/2}$

so $\ln 2 = \lambda t_{1/2}$

$$\boxed{\text{so } t_{1/2} = \frac{\ln 2}{\lambda}}$$

← very useful

remember ln 2 is just a number → 0.693....

so $t_{1/2} = \dfrac{0.693...}{\lambda}$