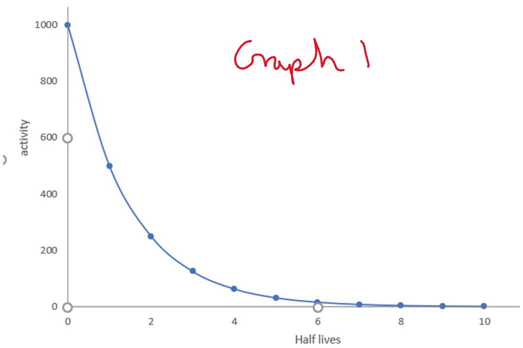
Radioactivity 005

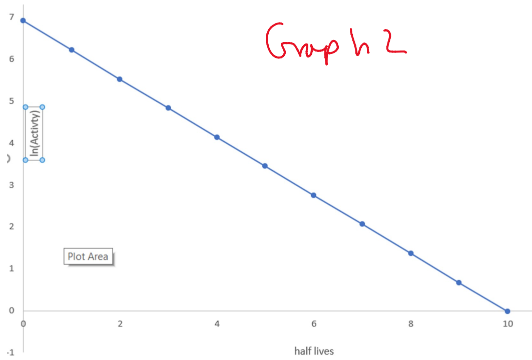
14 November 2020   10:28

The maths of exponential and logs can be confusing. This sheet should help.

The first graph is easy - its just a plot of time (measured in half lives) and activity on the y axis. You get a clear curve known as an 'exponential decay curve'.

What is helpful is that you get exactly the same shape of curve for radioactive decay and the graphs for discharging of a capacitor. In other words the maths is the same which governs capacitors and radioactivity. That is genuinely awesome.





What is the second graph?

Well, first and foremost it is a straight line!

A table like this was to create the top graph:

Number of half lives	Activity
0	1000
1	500
2	250
3	125
5	62.5

Now we will add a third column to our table. This will be ln(activity) - take 'natural logs of the activity value'. If we plot the ln (activity) we get the straight line meaning that we can use

y=mx+c

Number of half lives	Activity		Ln(activity) value
0	1000	Ln(1000)	6.91
1	500	Ln(500)	6.21
2	250	Ln(250)	5.52
3	125	etc	4.83
5	62.5	etc	4.14

Now what is the gradient of the ln(A) vs half lives graph equal to?

To do this we need to work with the general decay formula and 'get rid' of the exponent, e.
We do this by taking logs.

$$V = V_0 e^{-\frac{t}{RC}}$$

$$A = A_0 e^{-\lambda t}$$

This is the formula for radioactive decay. It is very like the one for v on a capacitor as it discharges

take logs:

$$\ln(A) = \ln(A_0 e^{-\lambda t})$$

$$\ln(AB) = \ln(A) + \ln(B)$$

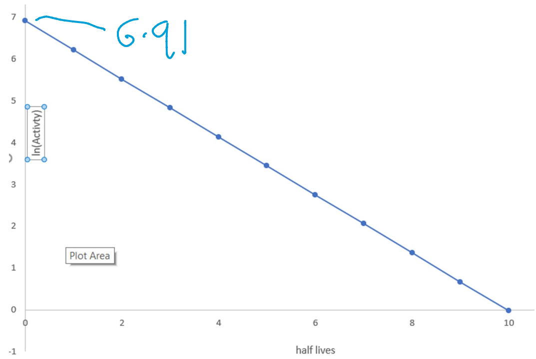
$$\ln(A) = \ln A_0 + \ln(e^{-\lambda t})$$

$$\ln(e^{x}) = x$$

so

$$\ln(A) = \ln A_0 + -\lambda t$$

$$\ln(A) = \ln(A_0) - \lambda t$$



We know that y=mx+c

Looking at the y axis we have ln(A). So y in y=mx+c is Ln(A)

So we could write:

Ln(A)=mx+c. Next question - what does the x equate to?

Looking at the ln graph we can see that half lives is on the x axis. So in y=mx+c the variable x is 'half lives'. Remember too that half lives are measure in time - so x can also be thought of as time, t.

So we can say Ln(A) = mt+c

That leaves m and c - what are they.

Well c is the 'y-intercept' - looking on the graph this is '6.91' - but take care - this does not mean A=6.91 but rather ln(A)=6.19

Therefore the actual value of A is $e^{6.91}$ which is 1002 (ie 1000!)

Lastly what is M - the gradient. So what does the gradient equate to?

$$y = mx + c$$

$$\ln(A) = \ln(A_0) - \lambda t$$

We have seen that y = ln(A) and x=half lives = t

In y = mx+c x is multiplied by m, the gradient. SO what is y multiplied by - that will be the gradient. IN this case it is $-\lambda$ please note the minus sign.

Lastly, that leaves C - which is the Y intercept which we saw was 1000.

So the gradient is the negative of decay constant (λ)