

Radioactivity 006 Modelling Iterative

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Activity A = number of unstable nuclei that decay each second

Decay constant λ = probability that a given nucleus will decay in a time interval (usually per second)

N = number of nuclei

$$A = \lambda N$$

If you plot the remaining, undecayed nuclei vs time you get a decay curve

Many quantities behave in this manner: the rate of change of the quantity depends on the amount of that quantity - you can see this as the curve is less steep as there are fewer nuclei left. This change is known as 'differential' equation and can be written like this:

$$\frac{dN}{dt} \propto -N \text{ and with the constant } \lambda$$

$$\frac{dN}{dt} = -\lambda N \dots \text{eqn 1}$$

This has the exact solution: $N = N_0 e^{-\lambda t}$

With this exact solution we can find the number of remaining nuclei at any value of t - which is obviously fab. However, many differential equations do not have exact solutions, so we need a different approach - the iterative one.

Rearrange (1)

$$dN = -\lambda N dt$$
$$\Rightarrow \Delta N = -\lambda N \Delta t$$

$\Delta t \sim$ because not infinitesimally small

Keep going :)

Now it's best to use an example:

Suppose we start with 10,000 nuclei. The decay constant is 0.2 and we will use a time interval of 0.1 seconds. How many nuclei are left after 0.5 seconds? You decide the time interval. Now its about working out how many decays you get in that time interval, and then subtracting that from the number of nuclei at the start of that interval.

t/s	ΔN	N
0	N/A	10,000
0.1	$-0.2 \times 10,000 \times 0.1 = -200$	$10,000 - 200 = 9,800$
0.2	$-0.2 \times 9800 \times 0.1 = -196$	$9800 - 196 = 9604$
0.3	$-0.2 \times 9604 \times 0.1 = -192.08$	$9604 - 192 = 9412$
0.4	$-0.2 \times 9412 \times 0.1 = -188.24$	$9412 - 188 = 9224$
0.5	$-0.2 \times 9224 \times 0.1 = -184.48$	$9224 - 185 = 9039$