Gravitational Fields - Satellites & Kepler's Laws

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Kepler came up with several laws about how any object orbiting a star (for example) behaves. One of these you will come across - it is Kepler's Third Law. In this he linked centripetal force and the gravitational force between the planet and the sun. In fact, he didn't just link them - he said they were the same.

Centripetal force $= \frac{mv^2}{r}$

Beware! The m here is the mass of the thing doing the rotating ~ ie the planet.

Gravitational force between the star (M) & the planet (m)

$$F = \frac{GMm}{r^2}$$

Setting them equal:

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

$$\Rightarrow \frac{GM}{r} = v^2$$

$$\Rightarrow v = \sqrt{\frac{GM}{r}} \quad \ldots \text{(1)} \quad \Rightarrow v \sqrt{\frac{1}{r}}$$

Equation 1 tells us the speed of an object in orbit - and further that it doesn't matter what the mass of the orbiting object it - the v only depends on r^{-1}.

On we go... Kepler was interested in T, the period, ie how long to orbit once round the star... Next recall $v = \frac{d}{t}$ and that if we assume a circular orbit (which is wrong...) then $d = 2\pi r$ for one orbit... ie $t =$ for 1 orbit is T → the period.

get another physics approximation

So $v = \frac{2\pi r}{T}$ .... ⇒ $T = \frac{2\pi r}{v}$ .... (2)

Now sub (1) ⇒ (2)

$$T = \frac{2\pi r}{\sqrt{\frac{GM}{r}}}$$ .... going slowly on the algebra:

$$\Rightarrow T = \frac{2\pi r}{\left(\frac{\sqrt{GM}}{\sqrt{r}}\right)}$$

beware: the square root in the top version has to be together (hence bracket)

$$\Rightarrow T = \frac{2\pi r\sqrt{r}}{\sqrt{GM}}$$

oh... nasty.... square everything.

$$\Rightarrow T^2 = \frac{4\pi^2 r^2 r}{GM} \Rightarrow \boxed{T^2 = \frac{4\pi^2 r^3}{GM}}$$

So ...

$T^2 \propto r^3$

orbital period radius of orbit

What is exciting about this formula is that it is full of constants.... G, M, 4, pi....

It tells us that the time for an orbit only depends on the radius - and nothing else

What is also amazing is that Kepler could check this out by observing the 5 visible planets given that he died in 1630 .....

So - it doesn't matter what size a planet it - its orbital period is only dictated by the M of the star, and the orbit's radius. You can also see that if a planet closer to the sun, eg Venus moves faster than one which is further - like Mars.

Other Things About Satellites

If the orbit is perfectly circular, and there is no loss of energy, then the PE and KE stay fixed. However, most orbits are elliptical - and so the value of r varies during the orbit. So as a satellite get's nearer it loses PE, gains KE and thus moves faster. (Obviously things are different if we have an energy source)

Geostationary Orbits - these are when a satellite takes the same time to orbit as the planet takes to rotate ones. We say they are in synchronous orbit. They need to be directly above the equator, have a radius of 42000Km and have the same angular speed as the earth. Low orbit satellites are below 2000Km above the earth surface. These are easier to get into space obviously, and tend to pass over the poles. Since their period is not the same as the planet's they will 'scan' over the whole surface over a period of time.

There are many artificial satellites orbiting earth--- this raises the interesting, and mostly ignored question of 'space junk'...