Turning Points - Special Relativity

20 March 2020 14:23

This is obviously only meant to whet the appetite...

Time Dilation

Time seems to go at different speeds for different observers.
Remember that when we say stationary we don't really mean not moving .... We can't tell is we're moving at a constant speed, or are stationary. When we say 'stationary observer' we mean, an observe that is moving at the same speed as the thing that is being observed.

A stationary observer measure the time between two events. This is the proper time, t_0.
An observer moving at a steady speed will measure a longer time interval t.

This is called 'time dilation' and is proved (well, as much as anything is...)

$t = \dfrac{t_0}{\sqrt{1 - \dfrac{v^2}{c^2}}}$	Since it crops up a lot we define γ (gamma) as the Lorentz factor. $\gamma = \dfrac{1}{\sqrt{1 - \dfrac{v^2}{c^2}}}$

Muon's Stick up for Time Dilation

Created in the upper atmosphere and moving towards the earth a speed of 099c.
At rest they have a half life of less than 1.53μs.

So ... Measure the count rate high up in the atmosphere. Then do the same near the ground. Given the data you can predict what they count rate at the top should be - you know how long they take to get to the earth (v=d/t) and so you can work out what the rate should be. In fact the rate is much higher.

Say its 2000m from ground to the first detector height. So they take

$\dfrac{2000}{0.99c} = 6.73 \times 10^{-6}$ seconds.

This is $\dfrac{6.73\mu}{1.53\mu}$ half times $= 4.4$

But from the muon's respective time has dilated. t_0 is now the proper time from the muon's perspective :

$$t = \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}} \Rightarrow 6.73\times10^{-6} \times \sqrt{1-\frac{0.99^2}{1^2}} = t$$

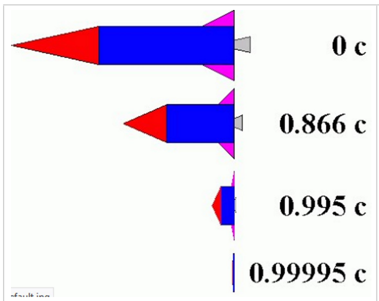
$$\Rightarrow t = 0.95\,\mu s$$

More Craziness - Length Contraction

An object moving in the same direction as its length looks short to an external observer (in an inertial frame of course...)

The length contraction forumla is very similar to the time dilation one... Those are small Ls...

$$l = l_0\sqrt{1-\frac{v^2}{c^2}}$$

	Suppose this rocket is 20m long when it is stationary cf to you. Suppose it is going at .9c - how long does it look to you? $L = 20 \times \sqrt{1 - \frac{0.9^2}{1^2}}$ $L = 8.7\,m$

Speed and Mass and Energy

The faster an object goes the more massive it becomes. You can see from the formula that as v gets close to c so m tends to ∞ hence no object (with rest mass) can go at c.

We can find the relativistic mass given the rest mass, m_o.

$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$	Einstein went on to derive $E=mc^2$ based on this eqn. This means there is equivalence between energy and mass. Put another way, if you increase an object's speed you increase its energy (not talking about E_k here though). The total energy in an object depends on its mass - so again, this goes to ∞ as c is approached.
$E = \frac{m_0C^2}{\sqrt{1-\frac{v^2}{C^2}}}$	Lastly, this gives the total energy of an object moving. It's another relativistic equation